Inspection Games with Local and Global Allocation Bounds

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Abstract: This article discusses a two-player noncooperative nonzero-sum inspection game. There are multiple sites that are subject to potential inspection by the first player (an inspector). The second player (potentially a violator) has to choose a vector of violation probabilities over the sites, so that the sum of these probabilities do not exceed one. An efficient method is introduced to compute all Nash equilibria parametrically in the amount of resource that is available to the inspector. Sensitivity analysis reveals nonmonotonicity of the equilibrium utility of the inspector, considered as a function of the amount of resource that is available to it; a phenomenon which is a variant of the well-known Braess paradox. Naval Research Logistics 60: 125–140, 2013

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1. INTRODUCTION

Inspection games model situations where an inspector verifies that one or more agents that are subject to inspection (denoted hereafter inspectees) follow some rules or regulations which were agreed on by all parties. Also, the inspector's verification process is defined in this agreement. Typically, the inspector has limited resources, so this verification can only be partial. A mathematical analysis of these games allows the understanding of inspection processes, where agents are strategic and rational. More importantly, the analysis of these games can also help with design of efficient inspection processes. Because the inspector and the inspectees (that is, the agents) each optimize its own utility, these processes should be modeled as games.

An extended summary of the literature on inspection games is available in Ref. [5]. Here, we will review a few of the two person inspection games that are related to this article, and some of the inspection games that have recently appeared in the literature.

In Ref. [9], Hohzaki et al. considered a (single-shot) multistage two-person zero-sum game. Each player has a finite number of opportunities to act, and it does not know the history of actions taken by its opponent. The inspector decides whether to inspect or not, and the inspectee decides whether to violate or not. The game ends when the inspectee is captured or when the preplanned period expires. Hohzaki et al. developed dynamic programming formulation to exhaust equilibrium points on a strategy space of each player.

In Ref. [8], Hohzaki considered a multistage two-person zero-sum stochastic game, where the inspectee decides how much contraband to smuggle. A closed-form equilibrium is derived for this specific case.

In Ref. [7], Haphuriwat et al. considered a two-person nonzero-sum sequential game. The effects of inspection and retaliation on the inspectee's decision were tested. Results show that unless the inspector imposes high retaliation costs on the inspectee, 100% inspection is likely to be needed, and deterrence with partial inspection may not be achievable in practice (even though it is possible in theory).

In Ref. [13], Xiaoqing et al. considered a two-person nonzero-sum game, where the inspectee is a (potentially) polluting firm. The authors explored the effects of subsidies, penalties, and other policy variables on implementation of cleaner production.

In Ref. [5], Deutsch et al. considered an inspection game between a single inspector whose inspection resource is constrained and multiple independent inspectees. The authors

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derived all Nash equilibria solutions for this game and discussed their properties.

This article focuses on a special case of the game formulated in Ref. [5]. Specifically, it considers a nonzero-sum inspection game between a single inspector and a single inspectee. There are multiple sites where the two players act, and both of them have limited resources. The model of Ref. [5] can be viewed as a two player game, where there is a "coordinator" that determines the actions of the multiple independent inspectees, and has sufficient resources to enable violation in all of the sites. Such a model can be used to describe many real-world scenarios, for example an owner of multiple (potentially polluting) plants, who decides which of its plants will comply with the pollution policies, and to what extent.

In this article, in addition to the global resource limitation, the inspector has to adhere to local limitations on the amounts of resource it can allocate to the inspection of the various sites. These additional local constraints make the analysis of the game more challenging. Nevertheless, we show how to compute all Nash equilibria solutions of this game in an efficient way. Sensitivity analysis of the Nash equilibria with respect to the inspector's available resource reveals a counterintuitive phenomenon which is a variant of the Braess Paradox Ref. [4]: the inspector's payoff function is not monotonically increasing in its resource.

Two-person nonzero-sum inspection games have been modeled using different approaches in previous work. In Ref. [11], the game has a finite number of "suspicious events," where the inspectee can violate. The inspectee can violate only once. The inspector may inspect a finite number of times, and it chooses which events to inspect. Also, the inspector can announce its strategy in advance, and may compensate the inspectee in case there is no violation. In Ref. [12], a similar model is introduced with a partition of the events to those in which the inspector can inspect and others in which it is not allowed to do so. In Ref. [3], the authors develop methods to inspect whether containers contain suspicious objects. They formulate an LP model to determine the optimal strategy of inspecting the contents of containers and deciding whether they are "good" or "bad."

In Ref. [2], there are two models related to the model of this article. The first model formulates a two-player inspection game where each one of the players picks one out of multiple sites. In the second model, the decisions of the inspector are different. The inspector's budget is given by an integer number (greater or equal to one), and it has to decide how much resource to allocate to each site, where there are no limitations on the amounts of resource that can be allocated to each site. In this article, the amount of resource available to the inspector is a real number, and the inspector has to adhere to site-specific bounds when allocating its resource. The inspectee has to choose whether to violate the regulations or not. If it decides to violate, then it has to select a vector of violation probabilities over the sites, such that the sum of these probabilities do not exceed the value of one. Also unlike Ref. [2], we are interested in computing all Nash equilibria.

The article is organized as follows. Section 2 formulates the inspection game and defines Nash equilibria. Section 3 establishes the existence of Nash equilibria and provides efficiently computable explicit expressions for all of them. Section 4 conducts sensitivity analysis and demonstrates how the inspector's amount of resource affects the Nash equilibria solutions. The proofs of results in Sections 2–4 appear in the Appendix. Section 5 illustrates the results through a numerical example. Finally, Section 6 concludes with a summary and directions for future research.

2. MODEL FORMULATION

This article concerns an inspection game between an inspector and an inspectee, denoted I and V, respectively. There is a set $N \equiv \{1, \ldots, n\}$ of sites where the inspector and the inspectee may have conflicting interests. The inspectee has to choose a vector of violation probabilities (whose sum do not exceed one), and the inspector has to determine the allocation of its inspection resources over the sites to detect the violations.

When the inspector has access to unlimited resources at no cost or when it has no resources at all, the problem becomes trivial. So, the inspector is assumed to have a limited amount of inspection resource, $B > 0$. The inspector has to decide on the amounts $x_1, \ldots, x_n$ to be allocated, respectively, to the inspection of the sites, where each such amount is limited from above. So, the inspector has to select a vector from

$$X \equiv \left\{ x = (x_1, \ldots, x_n) \in \mathbb{R}^n : 0 \le x_i \le \alpha_i \ \text{ for } \ i \in N \right. \\ \left. \text{and} \ \sum_{i \in N} x_i \le B \right\}, \tag{1}$$

where $\alpha_1, \ldots, \alpha_n$ and B are given positive numbers that satisfy $\sum_{i \in N} \alpha_i \ge B$ (because otherwise the global resource limitation is redundant). On the other hand, the inspectee has to decide on a vector of violation probabilities whose sum is constrained not to exceed one, that is, it can violate with certainty in no more than a single site. Thus, the inspectee has to determine a vector from:

$$Y \equiv \left\{ y = (y_1, \ldots, y_n) \in \mathbb{R}^n : y_i \ge 0 \ \text{ for } \ i \in N \right. \\ \left. \text{and} \ \sum_{i \in N} y_i \le 1 \right\}. \tag{2}$$

Here, $y_i = 1$, $y_i = 0$ and $0 < y_i < 1$ mean, respectively, full violation, full compliance, and partial compliance at site i.

The utility functions of I and V depend on $(x, y) \in X \times Y$ and are expressed by:

$$U^I(x, y) \equiv -\sum_{i \in N} y_i(d_i - c_i x_i), \tag{3}$$

and

$$U^V(x, y) \equiv \sum_{i \in N} y_i(a_i - b_i x_i), \tag{4}$$

respectively, where the quintuples $(a_i, b_i, c_i, d_i, \alpha_i)$ for $i \in N$ are given and the α_i's satisfy

$$\alpha_i \leq \frac{d_i}{c_i}, \quad \text{for all } i \in N, \tag{5}$$

the latter expresses the assumption that no matter what amount of resource the inspector deicides to invest, it will never cause its utility function as defined in (3) to be positive. Also, the confrontation at each site should cause nonnegative profit to the inspectee. However, we can assume (and we will demonstrate) that when the return from violation in site $i \in N$ is negative, then the inspectee decides to cooperate, so a similar assumption for α_i and $\frac{a_i}{b_i}$ for each $i \in N$ is unnecessary. The parameter a_i represents the inspectee's incentive to violate in site i, the parameter b_i represents the inspectee's penalty from violating in site i if inspected, the parameter d_i represents the inspector's penalty for not inspecting site i, and the parameter c_i represents the inspector's incentive to inspect site i.

A joint set of actions $(x^*, y^*) \in X \times Y$ is a Nash equilibrium if no player can benefit from deviating unilaterally from its strategy, that is,

$$U^I(x^*, y^*) = \max_{x \in X} U^I(x, y^*) \tag{6}$$

and

$$U^V(x^*, y^*) = \max_{y \in Y} U^V(x^*, y). \tag{7}$$

We say that x^* is a best response to y^* if it satisfies (6) and similarly y^* is a best response to x^* if (7) is satisfied.

Let $\mathbb{R}_\oplus \equiv \{z \in \mathbb{R} : z \geq 0\}$, and $a, b, c, d, \alpha \in \mathbb{R}^n_\oplus$. Suppose $x^* \in X$. If $a = 0$, then every $y \in Y$ is an inspectee's best response. Otherwise, if $b = 0$ and $a_i > 0$ for some $i \in N$, then the inspectee's best response is $\sum_{j \in \arg\max_{i \in N}\{a_j\}} y_j = 1$ and $y_i = 0$ otherwise. If $c^T \alpha = 0$, then every $x \in X$ is a best response for the inspector. Further, if $\alpha = 0$ (regardless of c), then $x = 0$ is a best response for the inspector. To avoid such degenerate situations, it is assumed throughout that

$$a_i, b_i, c_i, d_i, \alpha_i > 0 \quad \text{for all } i \in N. \tag{8}$$

As Nash equilibria with the inspector's utility function given by (3) are invariant with respect to the d_i's, henceforth, inspector's utility function will be given by

$$\widehat{U}^I(x, y) \equiv \sum_{i \in N} y_i c_i x_i \quad \text{for all } (x, y) \in X \times Y, \tag{9}$$

and it will express the "reduced cost" for the inspector rather than cost. Throughout, we shall consider the game defined by (1),(2),(9),(4), (5) and (8).

The following proposition establishes the existence of Nash equilibria for the game.

PROPOSITION 1: The game has Nash equilibria.

The next section computes all Nash equilibria of the game.

3. DETERMINING ALL NASH EQUILIBRIA

This section computes all Nash Equilibria of the game defined by (1), (2), (9), (4), (5) and (8).

For convenience, it is assumed henceforth that the a_i's and the c_i's are, respectively, distinct, and the sites are ordered as follows:

$$a_1 > a_2 > \ldots > a_n > a_{n+1} \equiv 0. \tag{10}$$

When there is only one site, the inspectee will cheat with probability 1 if this site has a positive return and will comply with probability 1 if it has a negative return. Hence, to avoid the need to discuss degenerate situations, we also assume that $n \geq 2$. For each $i \in N$ and for all $z \in \mathbb{R}$ define $p_i(z) \equiv a_i - b_i z$ and $q_i(z) \equiv c_i z$. Under a fixed inspection strategy $x^* \in X$, the problem that the inspectee faces is the ordinary linear knapsack problem with profit coefficients $p_i(x_i^*)$'s, unit cost for each $i \in N$, and a total budget of a single unit. Suppressing the dependence of the $p_i(x_i^*)$'s on x^*, the inspectee's problem is expressed by

$$\max \sum_{i \in N} p_i y_i \tag{11a}$$

$$\text{s.t.} \sum_{i \in N} y_i \leq 1, \tag{11b}$$

$$y_i \geq 0 \quad \text{for each } i \in N. \tag{11c}$$

Further, for a given violation strategy $y^* \in Y$, $q_i(y_i^*)$ is the expected profit (reduced cost) of the inspector per unit invested in inspecting site $i \in N$. As $c_i > 0$ and $y_i^* \geq 0$, $q_i(y_i^*) \geq 0$ for all $y^* \in Y$. Suppressing the dependence of the $q_i(y_i^*)$'s on y^*, the problem that the inspector faces is then given by

$$\max \sum_{i \in N} q_i x_i \tag{12a}$$

$$\text{s.t.} \sum_{i \in N} x_i \leq B, \tag{12b}$$

$$0 \leq x_i \leq \alpha_i \quad \text{for each } i \in N, \tag{12c}$$

which is the well-known bounded knapsack problem (see Ref. [10]).

In the following, we will refer interchangeably to optimal solutions of (11) and (12) and best responses, respectively, to x^* and y^*.

Lemmas 1 and 2 record the solutions of (11) and (12), respectively. As their proofs are standard, they are omitted. Throughout, the empty sum is defined to be 0.

LEMMA 1 (Unbounded Knapsack): (11) has an optimal solution. Further, letting $M = \arg\max_{j \in N} p_j$, $y^* \in \mathbb{R}^n$ is optimal for (12) if and only if $y^* \geq 0$, $\sum_{i \in N \setminus M} y_i^* = 0$, and one of the following cases holds:

i. $\max_{j \in N} p_j > 0$ and $\sum_{i \in M} y_i^* = 1$. In this case, there exists a $w \in N$ such that $p_w = \max_{j \in N} p_j > 0$ and $y_w^* > 0$.
ii. $\max_{j \in N} p_j = 0$ and $0 \leq \sum_{i \in M} y_i^* \leq 1$. In this case, if $y_w^* > 0$ for some $w \in N$, then $p_w = \max_{j \in N} p_j = 0$. Further, if $p_j = 0$ for all $j \in N$ then every $y \in Y$ is optimal for (11).
iii. $\max_{j \in N} p_j < 0$ and $\sum_{i \in M} y_i^* = 0$.

LEMMA 2 (Bounded Knapsack): (12) has an optimal solution. Assume that $q_1 \geq q_2 \geq \ldots \geq q_n \geq 0$, let $\sum_{j=1}^{k-1} \alpha_j < B \leq \sum_{j=1}^{k} \alpha_j$ for some $k \in N$, and $E \equiv \{i \in N : q_i = q_k\}$. Then, $x^* \in \mathbb{R}^n_{\oplus}$ is optimal for (12) if and only if:

- for $i \in \{1, \ldots, k\} \setminus E$ $x_i^* = \alpha_i$,
- for $i \in \{k+1, \ldots, n\} \setminus E$ $x_i^* = 0$,

and

$$\sum_{i \in E} x_i^* \begin{cases} = B - \sum_{i \in \{1,\ldots,k\} \setminus E} \alpha_i & \text{if } q_k > 0, \\ \leq B - \sum_{i \in \{1,\ldots,k\} \setminus E} \alpha_i & \text{if } q_k = 0. \end{cases}$$

The next corollary of Lemma 2 records simple necessary conditions for optimality in (12).

COROLLARY 1: Suppose that x^* is optimal for (12), and for some $u \in N$: $q_u > 0$ and $x_u^* < \alpha_u$. Then $\sum_{j \in N} x_j^* = B$, and $x_k^* = 0$ for each $k \in N$ with $q_k < q_u$.

A goal of the inspector is to reduce the coefficients $p_i(\cdot)$'s to zero, which would induce the inspectee to fully comply. But, the inspector is subject to two types of constraints. The bound constraints (12c) restrict the amounts of resource that can be allocated to each individual site, whereas the budget constraint (12b) restricts the total amount of resource that can be spent on all sites. When $a_i - b_i\alpha_i = p_i(\alpha_i) \leq 0$ (that is, $\frac{a_i}{b_i} \leq \alpha_i$) for each $i \in N$, the bound constraints are redundant. The following two theorems establish necessary and sufficient conditions for a Nash equilibrium, distinguishing between the case where this condition is satisfied strictly and the case where it is not. Specifically, let $\nu_- \equiv \{i \in N : \frac{a_i}{b_i} < \alpha_i\}$, $\nu_0 \equiv \{i \in N : \frac{a_i}{b_i} = \alpha_i\}$, and $\nu_+ \equiv \{i \in N : \frac{a_i}{b_i} > \alpha_i\}$, then $\nu_+ = \emptyset$ implies that the bound constraints are redundant.

THEOREM 1: Suppose $\nu_+ = \emptyset$ ($\alpha_i \geq \frac{a_i}{b_i}$ for each $i \in N$). Necessary and sufficient conditions for (x^*, y^*) to be a Nash equilibrium along with the resulting payoffs are listed below for the four mutually exclusive collectively exhaustive cases

i. $B > \sum_{j \in N} \frac{a_j}{b_j}$ and

$$(x_i^*, y_i^*) = \begin{cases} \left(\dfrac{a_i}{b_i}, \rho_i\right) & \text{if } i \in \nu_0, \\ (\xi_i, 0) & \text{if } i \in \nu_-, \end{cases} \tag{13}$$

with $\frac{a_i}{b_i} \leq \xi_i \leq \alpha_i$ for $i \in \nu_-$, $\sum_{i \in \nu_-} \xi_i \leq B - \sum_{i \in \nu_0} \frac{a_i}{b_i}$, $\rho_i \geq 0$ for $i \in \nu_0$, $\sum_{i \in \nu_0} \rho_i \leq 1$. The corresponding payoffs are:

$$\widehat{U}^I(x^*, y^*) = \sum_{i \in \nu_0} \rho_i c_i \frac{a_i}{b_i} \tag{14}$$

$$U^V(x^*, y^*) = 0.$$

ii. $B = \sum_{j \in N} \frac{a_j}{b_j}$, and for some $\zeta \geq 0$,

$$(x_i^*, y_i^*) = \begin{cases} \left(\dfrac{a_i}{b_i}, \rho_i\right) & \text{if } i \in \nu_0 \\ \left(\dfrac{a_i}{b_i}, \dfrac{\zeta}{c_i}\right) & \text{if } i \in \nu_-, \end{cases} \tag{15}$$

with $\rho_i \geq \frac{\zeta}{c_i}$ for $i \in \nu_0$, $\sum_{i \in \nu_0} \rho_i + \sum_{i \in \nu_-} \frac{\zeta}{c_i} \leq 1$. The corresponding payoffs are:

$$\widehat{U}^I(x^*, y^*) = \sum_{i \in \nu_0} \rho_i c_i \frac{a_i}{b_i} + \sum_{i \in \nu_-} \zeta \frac{a_i}{b_i} \tag{16}$$

$$U^V(x^*, y^*) = 0.$$

iii. $\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j} < B < \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for some $k \in N$, and

$$(x_i^*, y_i^*) = \begin{cases} \left(\frac{a_i - \delta}{b_i}, \frac{\frac{1}{c_i}}{\sum_{j=1}^{k} \frac{1}{c_j}} \right) & \text{if } i = 1, \dots, k \\ (0, 0) & \text{if } k < i \le n, \end{cases} \tag{17}$$

with $\delta \equiv \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - B}{\sum_{j=1}^{k} \frac{1}{b_j}}$. The corresponding payoffs are:

$$\widehat{U}^I(x^*, y^*) = \frac{B}{\sum_{j=1}^{k} \frac{1}{c_j}} \tag{18}$$
$$U^V(x^*, y^*) = \delta.$$

iv. $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for some $k \in \{1, \dots, n-1\}$, and

$$(x_i^*, y_i^*) = \begin{cases} \left(\frac{a_i - a_{k+1}}{b_i}, \frac{\frac{1-\eta}{c_i}}{\sum_{j=1}^{k} \frac{1}{c_j}} \right) & \text{if } i = 1, \dots, k \\ (0, \eta) & \text{if } i = k+1 \\ (0, 0) & \text{if } k+1 < i \le n, \end{cases} \tag{19}$$

for some $0 \le \eta \le \frac{1}{\sum_{j=1}^{k+1} \frac{c_{k+1}}{c_j}} (< 1)$. The corresponding payoffs are:

$$\widehat{U}^I(x^*, y^*) = \frac{(1-\eta)B}{\sum_{j=1}^{k} \frac{1}{c_j}} \tag{20}$$
$$U^V(x^*, y^*) = a_{k+1}.$$

The next result complements Theorem 1 by considering the case where $\nu_+ \neq \emptyset$. In this case, let $\tau \equiv \max_{i \in N}\{p_i(\alpha_i) = a_i - b_i\alpha_i\}$ (note that $\nu_+ \neq \emptyset$ implies $\tau > 0$), $\mu \equiv \max\{i \in N : a_i \ge \tau\}$, $\widehat{N} \equiv \{1, \dots, \mu\}$, $\widehat{\nu}_- \equiv \{i \in \widehat{N} : \frac{a_i - \tau}{b_i} < \alpha_i\}$, and $\widehat{\nu}_0 \equiv \{i \in \widehat{N} : \frac{a_i - \tau}{b_i} = \alpha_i\}$ (note that $\tau = a_i - b_i\alpha_i$ for some $i \in N$, that is, $\widehat{\nu}_0 \neq \emptyset$). For $i \in N$, $\tau \ge a_i - b_i\alpha_i$, that is, $\alpha_i \ge \frac{a_i - \tau}{b_i}$; hence, $\widehat{\nu}_+ \equiv \{i \in \widehat{N} : \frac{a_i - \tau}{b_i} > \alpha_i\} = \emptyset$ and $\widehat{\nu}_-$ and $\widehat{\nu}_0$ partition $\widehat{N}$. Further, for each $i \in N \setminus \widehat{N}$, $p_i(x_i) \le a_i < \tau$.

THEOREM 2: Suppose $\nu_+ \neq \emptyset$. Necessary and sufficient conditions for (x^*, y^*) to be a Nash equilibrium along with the resulting payoffs are listed below for the three mutually exclusive collectively exhaustive cases:

i. $B > \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$, and

$$(x_i^*, y_i^*) = \begin{cases} \left(\frac{a_i - \tau}{b_i}, \rho_i \right) & \text{if } i \in \widehat{\nu}_0 \\ (\xi_i, 0) & \text{if } i \in \widehat{\nu}_- \\ (\xi_i, 0) & \text{if } i \in N \setminus \widehat{N}, \end{cases} \tag{21}$$

with $\frac{a_i - \tau}{b_i} \le \xi_i \le \alpha_i$ for $i \in \widehat{\nu}_-$, $0 \le \xi_i \le \alpha_i$ for $i \in N \setminus \widehat{N}$, $\sum_{i \in \widehat{\nu}_0} \frac{a_i - \tau}{b_i} + \sum_{i \in \widehat{\nu}_-} \xi_i + \sum_{i \in N \setminus \widehat{N}} \xi_i \le B$, $\rho_i \ge 0$ for $i \in \widehat{\nu}_0$, $\sum_{i \in \widehat{\nu}_0} \rho_i = 1$. The corresponding payoffs are:

$$\widehat{U}^I(x^*, y^*) = \sum_{i \in \widehat{\nu}_0} \rho_i c_i \left(\frac{a_i - \tau}{b_i} \right) \tag{22}$$
$$U^V(x^*, y^*) = \sum_{i \in \widehat{\nu}_0} \rho_i \tau.$$

ii. $B = \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$ and for some $\zeta \ge 0$,

$$(x_i^*, y_i^*) = \begin{cases} \left(\frac{a_i - \tau}{b_i}, \rho_i \right) & \text{if } i \in \widehat{\nu}_0 \\ \left(\frac{a_i - \tau}{b_i}, \frac{\zeta}{c_i} \right) & \text{if } i \in \widehat{\nu}_- \setminus \{\mu\} \text{ or } i = \mu \text{ and } a_\mu > \tau \\ (0, \rho_\mu) & \text{if } i = \mu \in \widehat{\nu}_- \text{ and } a_\mu = \tau \\ (0, 0) & \text{if } i \in N \setminus \widehat{N}, \end{cases} \tag{23}$$

with $\rho_i \ge \frac{\zeta}{c_i}$ for $i \in \widehat{\nu}_0$, $\rho_\mu \le \frac{\zeta}{c_\mu}$ if $\mu \in \widehat{\nu}_-$ and $a_\mu = \tau$. The corresponding payoffs are:

If $i = \mu$ and $a_\mu > \tau$:

$$\widehat{U}^I(x^*, y^*) = \sum_{i \in \widehat{\nu}_0} \rho_i c_i \left(\frac{a_i - \tau}{b_i} \right) + \sum_{i \in \widehat{\nu}_- \setminus \{\mu\}} \zeta \left(\frac{a_i - \tau}{b_i} \right) + \zeta \left(\frac{a_\mu - \tau}{b_\mu} \right) \tag{24}$$
$$U^V(x^*, y^*) = \sum_{i \in \widehat{\nu}_0} \rho_i \tau + \sum_{i \in \widehat{\nu}_- \setminus \{\mu\}} \frac{\zeta}{c_i} \rho + \frac{\zeta}{c_\mu} \tau.$$

If $i = \mu \in \widehat{\nu}_-$ and $a_\mu = \tau$:

$$\widehat{U}^I(x^*, y^*) = \sum_{i \in \widehat{\nu}_0} \rho_i c_i \left(\frac{a_i - \tau}{b_i} \right) + \sum_{i \in \widehat{\nu}_- \setminus \{\mu\}} \zeta \left(\frac{a_i - \tau}{b_i} \right) \tag{25}$$
$$U^V(x^*, y^*) = \sum_{i \in \widehat{\nu}_0} \rho_i \tau + \sum_{i \in \widehat{\nu}_- \setminus \{\mu\}} \frac{\zeta}{c_i} \tau + \rho_\mu \tau.$$

iii. $0 < B < \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$, and (x^*, y^*) is a Nash equilibrium for the model where each α_i is replaced by $\bar{\alpha}_i \equiv \max\{\alpha_i, \frac{a_i}{b_i}\}$, to which Theorem 1 applies.

4. SENSITIVITY OF NASH EQUILIBRIA TO BUDGET CHANGES

The purpose of this section is to explore the effect of the changes in the amount B that is available to the inspector on the Nash equilibria. To do so, we consider "imperfect information" variant of the game we solved. In the modified game, each player knows its and the opponent's possible actions and payoff functions, but B is an exogenous parameter, which is unknown to the players. We assume that the players have some beliefs about B's value, but are uncertain about it.

We will consider the case where $\nu_+ = \emptyset$. The values $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$, for $k = 0, 1, \ldots, n$, will be referred to as singular amounts of the inspector's resource ($k = n$ corresponding to $B = \sum_{j \in N} \frac{a_j}{b_j}$). These singular amounts are treated distinctly in Theorem 1. The set of nonsingular amounts is the union of disjoint open intervals $I_k \equiv (\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j}, \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j})$, $k = 1, \ldots, n$; the closure of each I_k will be denoted $\bar{I}_k$.

The case where $\nu_+ = \emptyset$ is addressed in Theorem 1 which demonstrates: (i) for nonsingular amounts $B < \sum_{j \in N} \frac{a_j}{b_j}$, Nash equilibrium allocations of the inspector and the inspectee are unique, (ii) for singular amounts, Nash equilibrium allocations of the inspector are unique whereas the inspectee has multiple Nash equilibrium allocations, and finally, (iii) for $B > \sum_{j \in N} \frac{a_j}{b_j}$, Nash equilibrium allocations of both players are not unique. In either case, the Nash equilibrium allocations of the players are in product form, that is, their selection (by the inspector and by the inspectee) are independent. Consequently, the notion equilibrium strategies for the inspector and for the inspectee will be used. Further, the sets of equilibrium strategies will be indexed by B, using the notation $x^*(B)$ and $y^*(B)$, respectively. Formally, $x^*(\cdot)$ and $y^*(\cdot)$ express point to set mappings, but we refer to them as functions on subsets of their domain on which the corresponding ranges consist of singletons. In addition, the strategy sets will also be indexed by B, that is, $X(B)$ and $Y(B)$. Properties of $x^*(\cdot)$ and $y^*(\cdot)$ are next recorded. The symbol $\pm$ is used to indicate either $+$ or $-$.

LEMMA 3: Assume that $\nu_+ = \emptyset$.

i. For $B > \sum_{j \in N} \frac{a_j}{b_j}$, $y^*(B) = \{y \in Y(B) : y_i = 0 \text{ for each } i \in \nu_- \text{ and } y_i \geq 0 \text{ for each } i \in \nu_0\}$, and $x^*(B) = \{x \in X(B) : \frac{a_i}{b_i} \leq x_i \leq \alpha_i \text{ for each } i \in N\}$,
ii. $x^*(\cdot)$ is a piecewise linear, continuous and weakly increasing function on the interval $(0, \sum_{j \in N} \frac{a_j}{b_j}]$. Further, for $i \in N$, $x_i^*(\cdot)$ is zero on $(0, \sum_{j=1}^{i-1} \frac{a_j - a_i}{b_j}]$ and for $i \leq k \leq n$, its slope on I_k is $\frac{\frac{1}{b_i}}{\sum_{j=1}^{k} \frac{1}{b_j}}$, which decreases in $i \leq k \leq n$, so, $x_i^*(\cdot)$ is concave on $[\sum_{j=1}^{i-1} \frac{a_j - a_i}{b_j}, \sum_{j=1}^{n} \frac{a_j}{b_j}]$.
iii. For $i \in N$, $y_i^*(\cdot)$ is zero on $(0, \sum_{j=1}^{i-1} \frac{a_j - a_i}{b_j})$ and for $i \leq k \leq n$, $y_i^*(\cdot)$ equals $\frac{\frac{1}{c_i}}{\sum_{j=1}^{k} \frac{1}{c_j}}$ on I_k, a constant which decreases in k.
iv. For $i \in N$ and any singular amount B, let $y_i^*(B \pm 0) \equiv \lim_{\epsilon \downarrow 0} y_i^*(B \pm \epsilon)$. If $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for $k \in N \setminus \{n\}$, then

$$y_i^*(B) = \begin{cases} \left[y_i^*(B-0) = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k} \frac{1}{c_j}},\ y_i^*(B+0) = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k+1} \frac{1}{c_j}} \right] & \text{for } i = 1, \ldots, k \\ \left[y_i^*(B-0) = 0,\ y_i^*(B+0) = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k+1} \frac{1}{c_j}} \right] & \text{for } i = k+1 \\ \{0\} & \text{if } i = k+2, \ldots, n \end{cases} \tag{26}$$

The results of Lemma 3 are next used to determine the equilibrium utility functions of the inspector and of the inspectee. The notation $(\widehat{U}^I)^*(B)$ and $(U^V)^*(B)$ will be used for the set of equilibrium utility payoffs of the corresponding player under B, for example, $(\widehat{U}^I)^*(B) = \{\widehat{U}^I(x^*, y^*) : (x^*, y^*) \text{ is a Nash equilibrium when } B \text{ is the amount of available resource}\}$. As is done for the equilibrium strategies, we refer to $(\widehat{U}^I)^*(\cdot)$ and $(U^V)^*(\cdot)$ as functions on subsets of their domain on which the corresponding ranges consist of singletons.

THEOREM 3: Assume that $\nu_+ = \emptyset$.

i. For $B > \sum_{j \in N} \frac{a_j}{b_j}$, $(U^V)^*(B) = 0$ and $(\widehat{U}^I)^*(B) \geq 0$.
ii. $(U^V)^*(\cdot)$ is piecewise linear, continuous, decreasing and convex in B on the interval $(0, \sum_{j \in N} \frac{a_j}{b_j}]$, further, its slope on I_k is $\frac{-1}{\sum_{j=1}^{k} \frac{1}{b_j}}$; these negative constants increase in k.
iii. For $k \in N$, $(\widehat{U}^I)^*(\cdot)$ is linearly increasing in B on I_k with slope $\frac{1}{\sum_{j=1}^{k} \frac{1}{c_j}}$; these positive constants decrease in k.
iv. For singular amount $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$, let $(\widehat{U}^I)^*(B \pm 0) \equiv \lim_{\epsilon \downarrow 0} (\widehat{U}^I)^*(B \pm \epsilon)$. If $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for $k \in N$, then

$$(\widehat{U}^I)^*(B) = \left[(\widehat{U}^I)^*(B-0) = \frac{\sum_{j=1}^{k}(\frac{a_j - a_{k+1}}{b_j})}{\sum_{j=1}^{k}\frac{1}{c_j}}, \quad (\widehat{U}^I)^*(B+0) = \frac{\sum_{j=1}^{k}(\frac{a_j - a_{k+1}}{b_j})}{\sum_{j=1}^{k+1}\frac{1}{c_j}} \right] \tag{27}$$

Lemma 3, part (ii) and Theorem 3, parts (iii) and (iv), demonstrate a counterintuitive phenomenon: while the x_i^*'s are monotonically increasing and continuous in B, $(\widehat{U}^I)^*(\cdot)$ is not. In particular, $(\widehat{U}^I)^*(\cdot)$ increases on each (open) interval I_k of nonsingular amounts, and then it "drops" at the singular amounts, that is, $(\widehat{U}^I)^*(\cdot)$ is not monotonically increasing in B. At the following interval I_{k+1}, the slope of $(\widehat{U}^I)^*(\cdot)$ is still positive, but lower. This nonmonotonicity effect might be considered as a variant of the Braess paradox Ref. [4]: adding more resources to a network where the agents are strategic and rational does not necessarily increase their payoffs. Here, the resources are added to the inspector and affect negatively only its payoff. This variant of the Braess paradox occurs from a similar reason as the original one: noncooperative Nash equilibria are not necessarily Pareto efficient Ref. [6]. In these singular amounts, the inspectee is indifferent between violating in sites $1, \dots, k$ $(1 \leq k < n)$ and violating in sites $1, \dots, k+1$, while the inspector is determined to inspect only in sites $1, \dots, k$. Hence, when the inspectee decides to violate in the larger set of sites $(1, \dots, k+1)$, the inspector's payoff decreases. This observation means that for every selection of parameters for this game, there is a set of budget-values beyond which there are intervals of discontinuity in the budgets allocated to the inspector. In other words, the inspector's true payoff function can be depicted as a step function, for example, there is an interval of discontinuity in budgets allocated for the inspector at [5, 7.5] in the (following) Figure 3. That is, if the inspector is offered a budget within this interval, than it would prefer to stay with a budget of 5. This phenomenon is further demonstrated in the forthcoming numerical examples.

5. NUMERICAL EXAMPLES

EXAMPLE 1: Consider an example with the data given in Table 1 (as B is an exogenous parameter, it is not given as part of the game's data):

Table 1. Data for example 1.

i	a_i	b_i	c_i	α_i
1	30	1	10	50
2	25	1	20	30
3	20	1	15	40
4	18	1	5	18
5	12	1	17	17
6	10	1	12	36
7	7	1	11	75

In this example, there are seven sites. Also, $\alpha_i \geq \frac{a_i}{b_i}$ for each $i \in N$, so $\nu_+ = \emptyset$. This case is addressed in Theorem 1. In particular, the inspector has no local limitations [see (12c)], and it can inspect the sites such that their $p_i(x_i)$'s will have a value of 0. Nash equilibria solutions of this problem are depicted in Figures 1–4, parametericallly in B. We next offer some interpretations for these figures.

Figures 1 and 2 represent equilibrium values of x_i's and y_i's as a function of B, respectively. The inspector starts inspecting site 1, and continues to increase its allocation until $p_1(x_1^*)$ satisfies: $p_1(x_1^*) \equiv a_1 - b_1 x_1^* = a_2$. Under this allocation, the inspectee violates with certainty in site 1, that is, it invests $y_1^* = 1$ (as this site is the most beneficial to it), and complies with certainty in sites 2-7, that is, it invests 0 in them. At $B = 5$, sites 1 and 2 are equally beneficial for the inspectee, and it switches to a mode of partial violation at them, with $0.66 \leq y_1^* \leq 1$, $0 \leq y_2^* \leq 0.33$, and $y_i^* = 0$ for $i = 3, \dots, 7$. When $5 < B < 15$, the extra resource $B - 5$ of the inspector is allocated to sites 1 and 2, such that $p_1(x_1^*) = p_2(x_2^*) \geq a_3$. The inspectee partially violates in sites 1 and 2 with $y_1^* = 0.66$, $y_2^* = 0.33$, and complies with certainty at the other sites with $y_i^* = 0$ for $i = 3, \dots, 7$. When $B = 15$, sites 1,2 and 3 are equally beneficial for the inspectee, and hence it partially violates at these sites, with $0.46 \leq y_1^* \leq 0.66$, $0.23 \leq y_2^* \leq 0.33$, $0 \leq y_3^* \leq 0.3$, and $y_i^* = 0$ for $i = 4, \dots, 7$. When $15 < B < 21$, the extra resource $B - 15$ of the inspector is allocated to sites 1,2 and 3 in the same way, and so on until the point where $B = 122$. At this point, all sites are inspected such that their $p_i(x_i^*)$'s have a value of 0, and so the inspectee complies with certainty.

Figure 3 plots $\widehat{U}^I(x^*, y^*)$ as a function of B. The equilibrium utility of the inspector is expressed as a point-to-set function of the amount of resource B which exhibits "drops," at the points where the $p_i(x_i^*)$'s of the inspected sites have the same value as that of the next uninspected site, that is, at singular amounts of resource. This point to set map is linear and increasing between jumps, with decreasing slopes in progressing intervals.

Figure 4 expresses $U^V(x^*, y^*)$ as a function of B. The equilibrium utility of the inspectee is piecewise linear and decreasing.

EXAMPLE 2: Consider an example with the same data as in Table 1, except for α_4 and α_5, which now are changed into 8 and 7, respectively. Hence, $\alpha_4 < \frac{a_4}{b_4}$ and $\alpha_5 < \frac{a_5}{b_5}$, so $\nu_+ \neq \emptyset$.

The case where $\nu_+ \neq \emptyset$ is addressed in Theorem 2. In this example, $\tau \equiv \max_{i \in N} p_i(\alpha_i) = 10$, and $\mu = \{max_{i \in N} a_i \geq \tau\} = 5$, implying that $\widehat{N} = \{1, \dots, 5\}$. Hence, the maximum amount of resource the inspector may use is: $\sum_{i \in \widehat{N}} \frac{a_i - \tau}{b_i} = 55$. The inspector and the inspectee act in a similar way to that explained in Example 1. However, the

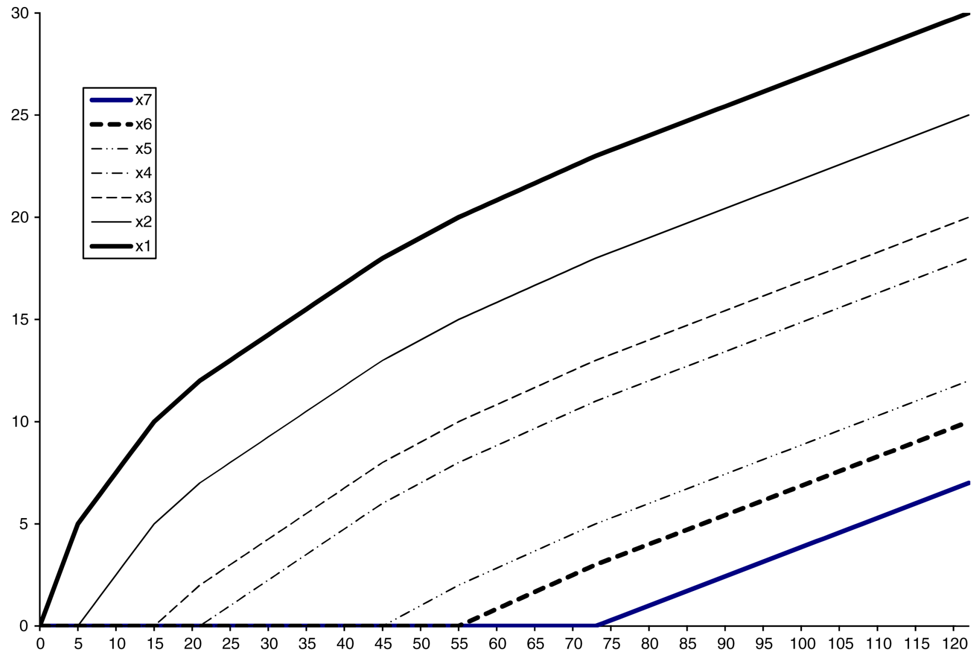

Figure 1. I's equilibrium strategies as a function of B when $\nu_+ = \emptyset$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

inspector cannot inspect beyond the point where the $p_i(x_i^*)$'s of sites 1–5 have the same positive value of 10, and so the inspectee is not deterred from violation.

The inspector's equilibrium values and the inspectee's equilibrium values are not plotted here, because they are almost identical to the relevant figures in Example 1, with

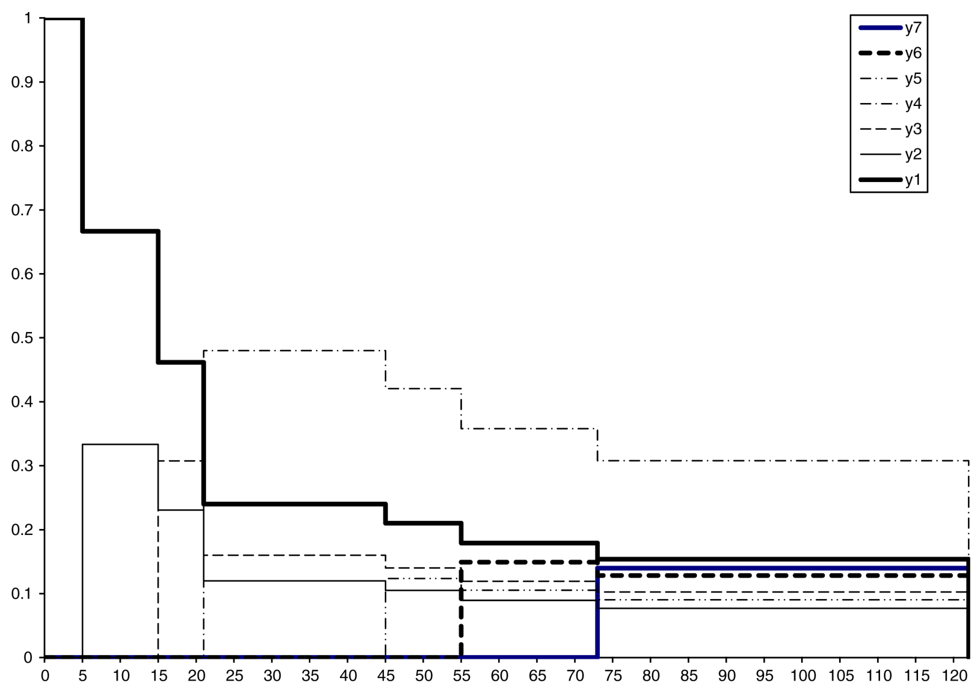

Figure 2. V's equilibrium strategies as a function of B when $\nu_+ = \emptyset$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

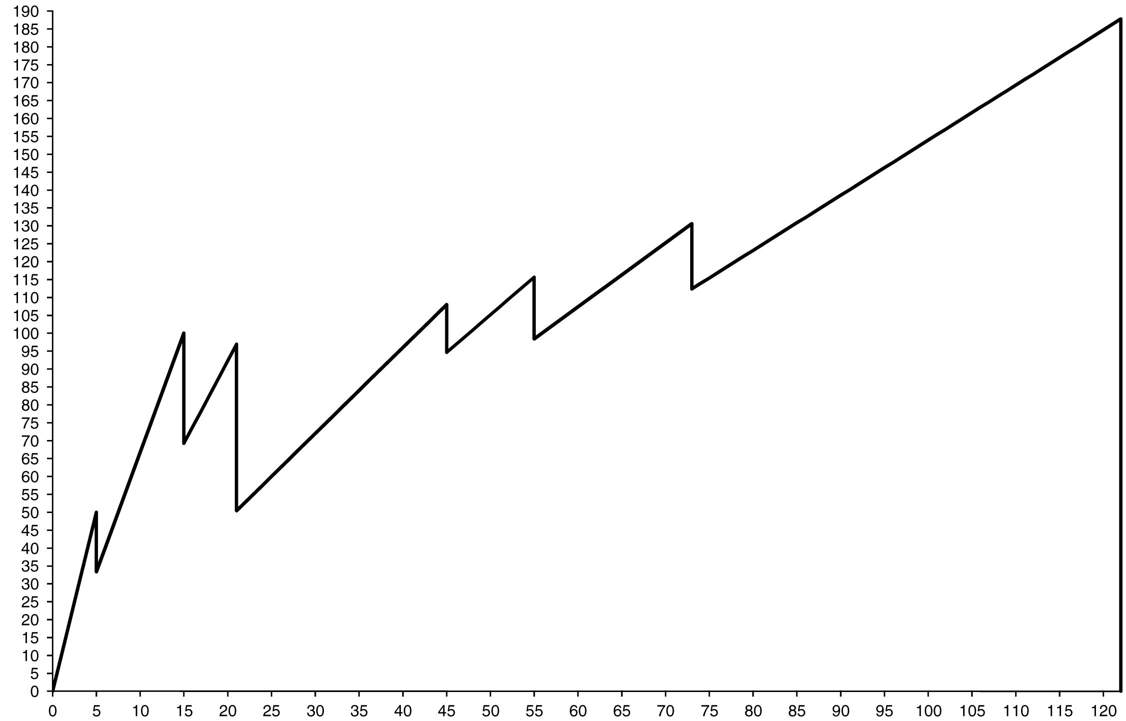

Figure 3. I's equilibrium payoffs as a function of B when $\nu_+ = \emptyset$.

the difference that the graphs are cut in the budget value of 55.

The inspector's payoff function and the inspectee's payoff function are depicted in Figures 5 and 6, parameterically in B.

6. CONCLUDING REMARKS

This article considers a (one stage) noncooperative nonzero-sum inspection game between an inspector and an inspectee who exhibit conflicting interests. Both players have

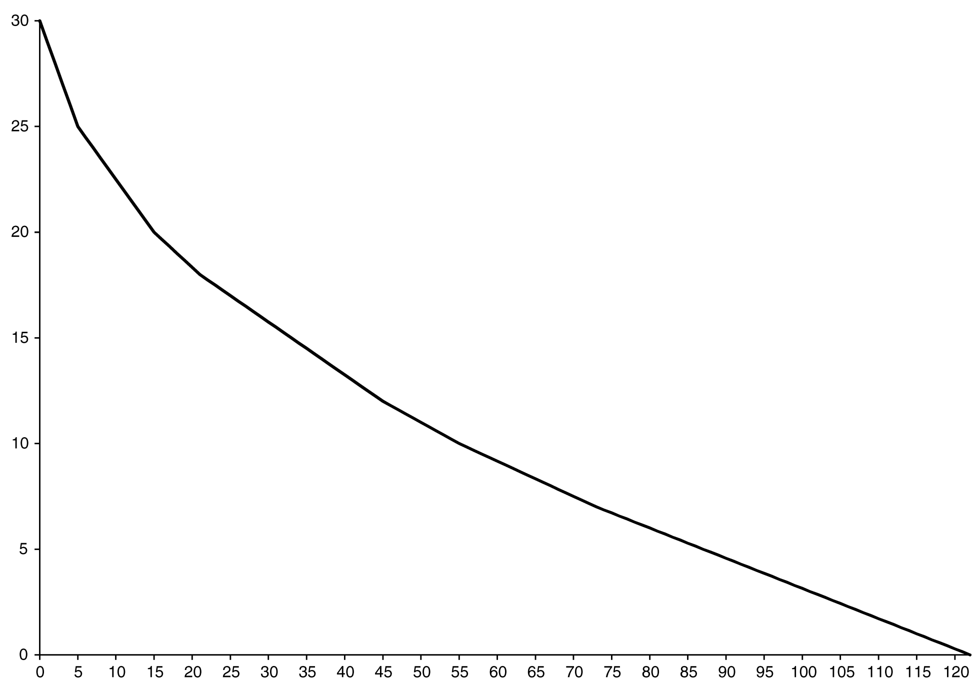

Figure 4. V's equilibrium payoffs as a function of B when $\nu_+ = \emptyset$.

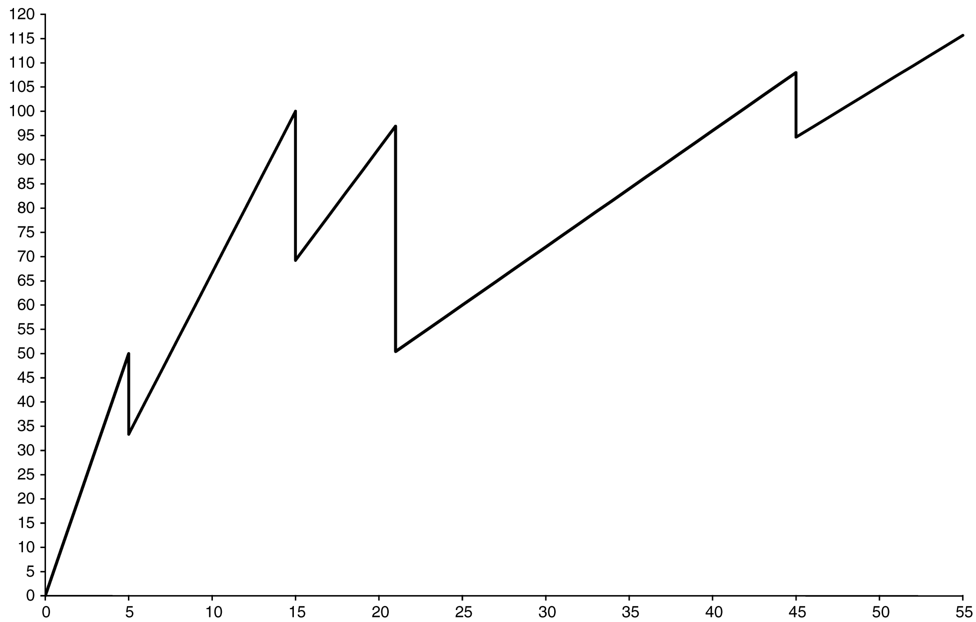

Figure 5. I's equilibrium payoffs as a function of B when $\nu_+ \neq \emptyset$.

limited resources available for their actions, and there are multiple sites where they can act. The inspectee has to choose a vector of violation probabilities over the sites (whose sum do not exceed one), and the inspector has to determine the allocation of its inspection resources over the sites to detect the violations. In addition to its global resource limitation, the inspector has local restrictions on the amounts of resource it can allocate to the sites. All Nash equilibria solutions are derived for this game.

In some cases, the structure of our game yields an interesting phenomenon where the inspector's utility decreases when the amount of resource available to it increases. Such cases

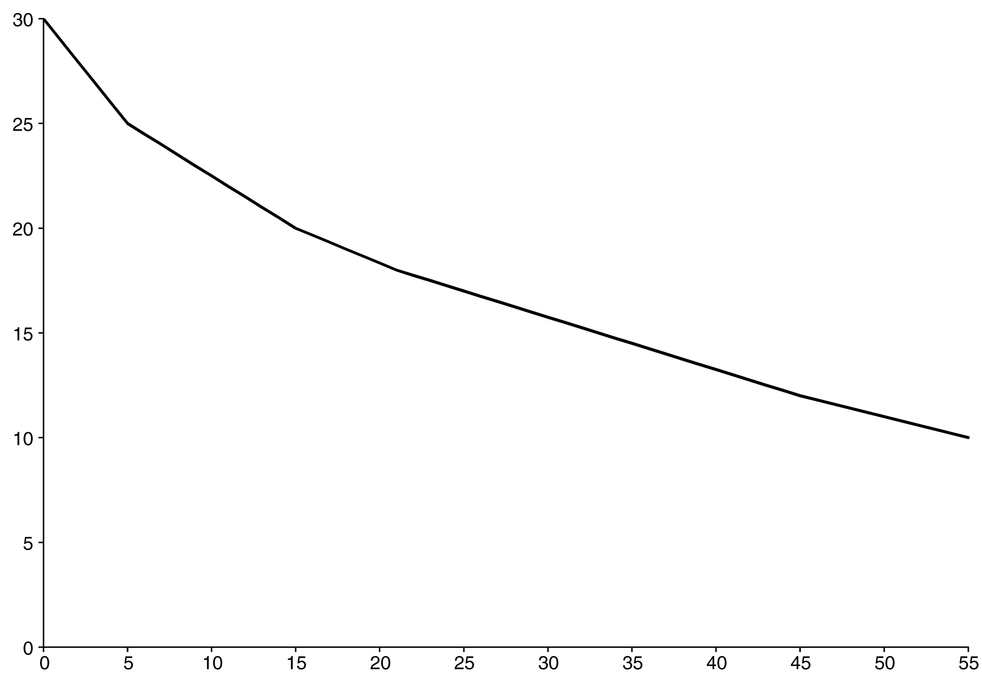

Figure 6. V's equilibrium payoffs as a function of B when $\nu_+ \neq \emptyset$.

were demonstrated in Sections 4 and 5, which also provide explanation as to what cause them.

In a follow-up work, which we have already begun, the game modeled here will be further extended to situations in which the inspectee can violate with certainty in more than one site. Further, investigating whether the counterintuitive phenomenon we observed here is still present in a repeated game of this model can also be of interest as future work.

APPENDIX A: PROOF OF PROPOSITION 1

As $\widehat{U}^I(x,y)$ and $U^V(x,y)$ are bilinear in (x,y), and X and Y are nonempty, convex and compact, the existence of Nash equilibria for the game follows from a classic result of Rosen Ref. [1]. ||

APPENDIX B: PROOF OF THEOREM 1

Sufficiency

i. $B > \sum_{j\in N} \frac{a_j}{b_j}$:

Assume that (x^*, y^*) satisfies (13) with corresponding ξ_i's and ρ_i's. Then, clearly, $x^* \in X$ and $y^* \in Y$. Also, $p_i(x_i^*) = a_i - b_i x_i^* \le 0$ for each $i \in N$, implying that a best response of the inspectee to x^* is any vector $y \in Y$ satisfying $y_i = 0$ for $i \in N$ with $p_i(x_i^*) < 0$ (Lemma 1(ii)). As $\{i \in N : p_i(x_i^*) < 0\} = \{i \in N : x_i^* > \frac{a_i}{b_i}\} = \{i \in N : \frac{a_i}{b_i} < x_i^* \le \alpha_i\} \subseteq \nu_- \subseteq \{i \in N : y_i^* = 0\}$, y^* is such a (best) response. On the other hand, if $y_i^* > 0$, then (by the definition of ν_0) $i \in \nu_0$ and $x_i^* = \frac{a_i}{b_i} = \alpha_i$. So, $x_i^* = \alpha_i$ whenever $q_i(y_i^*) = c_i y_i^* > 0$. By Lemma 2, x^* is a best response of the inspector to y^*.

ii. $B = \sum_{j\in N} \frac{a_j}{b_j}$:

Assume that $\zeta \ge 0$ and (x^*, y^*) satisfies (15) with corresponding ρ_i's. Then, clearly, $x^* \in X$ ($x_i^* = \frac{a_i}{b_i} \le \alpha_i$ for each $i \in N$ and $\sum_{j\in N} x_j^* = \sum_{j\in N} \frac{a_j}{b_j} = B$) and $y^* \in Y$. Next, observe that $p_i(x_i^*) = a_i - b_i x_i^* = 0$ for each $i \in N$, implying (by Lemma 1(ii) that any vector $y \in Y$ is a best response of the inspectee to x^*. In particular, y^* is such a response. On the other hand, $q_i(y_i^*) = c_i y_i^* = \zeta \ge 0$ for $i \in \nu_-$, and $q_i(y_i^*) = c_i y_i^* = c_i\rho_i \ge \zeta$ for $i \in \nu_0$. By Lemma 2, any $x \in X$ with $\sum_{i\in N} x_i = B$ and $x_i = \alpha_i$ for each i with $q_i(y_i^*) > \zeta$ is a best response of the inspector to y^*. As $c_i y_i^* = q_i(y_i^*) > \zeta$ implies that $i \in \nu_0$ and $x_i^* = \frac{a_i}{b_i} = \alpha_i$, x^* is such a response.

iii. $\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j} < B < \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for some $k \in N$:

Assume that (x^*, y^*) satisfies (17) with $\delta \equiv \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - B}{\sum_{j=1}^{k} \frac{1}{b_j}}$. Then, clearly, $y^* \in Y$. Also, as $\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j} < B < \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$,

$$\delta \equiv \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - B}{\sum_{j=1}^{k} \frac{1}{b_j}} > \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}}{\sum_{j=1}^{k} \frac{1}{b_j}} = a_{k+1} \ge 0,$$

and

$$\delta \equiv \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - B}{\sum_{j=1}^{k} \frac{1}{b_j}} < \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - \sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j}}{\sum_{j=1}^{k} \frac{1}{b_j}} = \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - \sum_{j=1}^{k} \frac{a_j - a_k}{b_j}}{\sum_{j=1}^{k} \frac{1}{b_j}} = a_k.$$

For $i = 1, \ldots, k$, it then follows that $\alpha_i \ge \frac{a_i}{b_i} > \frac{a_i - \delta}{b_i} = x_i^*$ and $x_i^* = \frac{a_i - \delta}{b_i} > \frac{a_i - a_k}{b_i} \ge 0$ (the last inequality by (10)). The definition of δ also assures that $\sum_{j\in N} x_j^* = \sum_{j=1}^{k} \frac{a_j - \delta}{b_j} = B$. So, $x^* \in X$. Next, observe that

$$p_i(x_i^*) = a_i - b_i x_i^* = \begin{cases} \delta & \text{for } i = 1, \ldots, k \\ a_i \le a_{k+1} < \delta & \text{for } i = k+1, \ldots, n, \end{cases}$$

implying (by Lemma 1(i)) that a best response of the inspectee to x^* is any vector $y \in Y$ satisfying $y_i = 0$ for $i = k+1, \ldots, n$ and $\sum_{j=1}^{k} y_j = 1$. In particular, y^* is such a response. On the other hand,

$$q_i(y_i^*) = c_i y_i^* = \begin{cases} \frac{1}{\sum_{i=1}^{k} \frac{1}{c_i}} & \text{for } i = 1, \ldots, k \\ 0 & \text{for } i = k+1, \ldots, n, \end{cases}$$

implying (by Lemma 2) that a best response of the inspector to y^* is any vector $x \in X$ satisfying $x_i = 0$ for $i = k+1, \ldots, n$ and $\sum_{j=1}^{k} x_j = B$. In particular, x^* is such a response.

iv. $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for some $k \in \{1, \ldots, n-1\}$:

Assume that (x^*, y^*) satisfies (19) with $0 \le \eta \le \frac{1}{\sum_{j=1}^{k+1} \frac{c_{k+1}}{c_j}}$. As $k < n$ (that is, c_{k+1} is defined), clearly, $\frac{1}{\sum_{j=1}^{k+1} \frac{c_{k+1}}{c_j}} < 1$. For each $i = 1, \ldots, k$, $x_i^* = \frac{a_i - a_{k+1}}{b_i} > 0$ (by (10)), and $\alpha_i \ge \frac{a_i}{b_i} > \frac{a_i - a_{k+1}}{b_i} = x_i^*$. Also, $\sum_{j\in N} x_j^* = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} = B$. So, $x^* \in X$. Next, as $0 \le \eta \le \frac{1}{\sum_{j=1}^{k+1} \frac{c_{k+1}}{c_j}} < 1$, it follows that $y_i^* \ge 0$ for each $i \in N$. Further, $\sum_{u\in N} y_u^* = \eta + \sum_{u=1}^{k} \frac{1-\eta}{c_u \sum_{j=1}^{k} \frac{1}{c_j}} = 1$. So $y^* \in Y$. Next, observe that

$$p_i(x_i^*) = a_i - b_i x_i^* = \begin{cases} a_{k+1} & \text{for } i = 1, \ldots, k+1 \\ a_i < a_{k+1} & \text{for } i = k+2, \ldots, n, \end{cases}$$

implying (by Lemma 1(i)) that a best response of the inspectee to x^* is any vector $y \in Y$ satisfying $y_i = 0$ for $i = k+2, \ldots, n$ and $\sum_{j=1}^{k+1} y_j = 1$. In particular, y^* is such a response. On the other hand,

$$q_i(y_i^*) = c_i y_i^* = \begin{cases} \frac{1-\eta}{\sum_{i=1}^{k} \frac{1}{c_i}} & \text{for } i = 1, \ldots, k \\ c_{k+1}\eta & \text{for } i = k+1 \\ 0 & \text{for } i = k+2, \ldots, n. \end{cases}$$

As $0 \le \eta \le \frac{1}{\sum_{j=1}^{k+1} \frac{c_{k+1}}{c_j}} = \frac{1}{1 + c_{k+1} \sum_{j=1}^{k} \frac{1}{c_j}}$, it follows that $0 \le \eta + c_{k+1}\eta \sum_{j=1}^{k} \frac{1}{c_j} \le 1$; so, $0 \le q_{k+1}(y_{k+1}^*) = c_{k+1}\eta \le \frac{1-\eta}{\sum_{j=1}^{k} \frac{1}{c_j}} = c_i y_i^* = q_i(y_i^*)$ and $q_i(y_i^*) = \frac{1-\eta}{\sum_{j=1}^{k} \frac{1}{c_j}} > 0$ for $i = 1, \ldots, k$. Thus (by Lemma 2), any vector $x \in X$ with $x_i = 0$ for $i = k+1, \ldots, n$ and $\sum_{i=1}^{k} x_i = B$ is a best response of the inspector to y^*. In particular, x^* is such a response.

Necessity

Suppose that (x^*, y^*) is a Nash equilibrium. The following five observations will be used to prove the necessity of cases (i)—(iv):

A. If $y^* = 0$, then (by Lemma 1(ii) and Lemma 1(iii)) for all $i \in N$, $p_i(x_i^*) \le 0$, that is, $x_i^* \ge \frac{a_i}{b_i}$.

B. If $y_u^* > 0$ for $u \in N$, then (by Lemma 1(i) and Lemma 1(ii)) $p_u(x_u^*) \ge 0$, that is, $x_u^* \le \frac{a_u}{b_u}$.

C. If $x_i^* < \frac{a_i}{b_i}$ for some $i \in N$, then $p_i(x_i^*) > 0$ and (by Lemma 1(i))

$$\{j \in N : y_j^* > 0\} \subseteq \arg\max_{j \in N} p_j(x_j^*)$$
$$\subseteq \{j \in N : p_j(x_j^*) > 0\} = \left\{ j \in N : 0 \le x_j^* < \frac{a_j}{b_j} \right\},$$

and

$$\sum_{j \in N} y_j^* = 1.$$

In particular, for some $w \in N$, $y_w^* > 0$ and $x_w^* < \frac{a_w}{b_w}$.

D. If $B < \sum_{j \in N} \frac{a_j}{b_j}$, then

$$\{i \in N : x_i^* > 0\} \subseteq \{i \in N : y_i^* > 0\}, \tag{28}$$

and

$$\sum_{i \in N} x_i^* = B. \tag{29}$$

Indeed, as $\sum_{j \in N} x_j^* \le B < \sum_{j \in N} \frac{a_j}{b_j}$, $x_k^* < \frac{a_k}{b_k}$ for some $k \in N$. It next follows from (C) that for some $w \in N$, $x_w^* < \frac{a_w}{b_w} \le \alpha_w$ and $y_w^* > 0$, the latter implying that $q_w(y_w^*) = c_w y_w^* > 0$. As $x_w^* < \alpha_w$ and $q_w(y_w^*) > 0$, (29) follows from Corollary 1. Further, if $y_u^* = 0$, then $q_u(y_u^*) = c_u y_u^* = 0 < q_w(y_w^*)$ and Corollary 1 implies that $x_u^* = 0$. So, $x_u^* > 0$ implies $y_u^* > 0$, verifying (28).

E. If $B < \sum_{j \in N} \frac{a_j}{b_j}$, $1 \le u < i$ and $y_i^* > 0$, then $y_u^* > 0$. Indeed, assume that $y_u^* = 0$. Then $q_i(y_i^*) = c_i y_i^* > 0 = c_u y_u^* = q_u(y_u^*)$ and (by Lemma 1(i) and Lemma 1(ii)) $p_u(x_u^*) \le p_i(x_i^*)$; in particular, $a_u - b_u x_u^* = p_u(x_u^*) \le p_i(x_i^*) \le a_i < a_u$ (the last inequality by (10)), implying that $x_u^* > 0$. As $q_i(y_i^*) > q_u(y_u^*)$ and $x_u^* > 0$, Lemma 2 implies that $x_i^* = \alpha_i \ge \frac{a_i}{b_i}$ and therefore $p_i(x_i^*) = a_i - b_i x_i^* \le 0$. As $y_i^* > 0$, Lemma 1(ii) implies that for each $j \in N$, $a_j - b_j x_j^* = p_j(x_j^*) \le p_i(x_i^*) \le 0$, that is, $x_j^* \ge \frac{a_j}{b_j}$ and therefore $\sum_{j \in N} \frac{a_j}{b_j} \le \sum_{j \in N} x_j^* \le B < \sum_{j \in N} \frac{a_j}{b_j}$, a contradiction.

i. $\underline{B > \sum_{j \in N} \frac{a_j}{b_j}}$:

It will be shown that (13) is satisfied with corresponding ξ_i's and ρ_i's. As $(x^*, y^*) \in X \times Y$, it suffices to show that $y_i^* = 0$ for $i \in \nu_-$ and $x_i^* \ge \frac{a_i}{b_i}$ for $i \in N$ (the latter would imply that for $i \in \nu_0$, $\frac{a_i}{b_i} \le x_i^* \le \alpha_i = \frac{a_i}{b_i}$, assuring that $x_i^* = \frac{a_i}{b_i}$). Assume first that $y_u^* > 0$ for some $u \in \nu_-$ and we will establish a contradiction. The assumption $y_u^* > 0$ implies that $q_u(y_u^*) = c_u y_u^* > 0$ and, by (B), $x_u^* \le \frac{a_u}{b_u} < \alpha_u$. Using Corollary 1, it then follows that $\sum_{j \in N} x_j^* = B > \sum_{j \in N} \frac{a_j}{b_j}$. Consequently, for some $v \in N \setminus \{u\}$, $x_v^* > \frac{a_v}{b_v}$, implying that $p_v(x_v^*) = a_v - b_v x_v^* < 0$ and (by Lemma 1(i)) $y_v^* = 0$. But, $q_u(y_u^*) > 0 = c_v y_v^* = q_v(y_v^*)$, $x_v^* > \frac{a_v}{b_v} > 0$, and $x_u^* < \alpha_u$ contradict Corollary 1. Next, assume that $x_u^* < \frac{a_u}{b_u} \le \alpha_u$ for some $u \in N$ and we will establish another contradiction. As $x_u^* < \frac{a_u}{b_u}$, (C) implies that for some $w \in N$, $y_w^* > 0$ and $x_w^* < \frac{a_w}{b_w} \le \alpha_w$. Replacing u by w in the above arguments leading to a contradiction under the assumption that $y_u^* > 0$ and $x_u^* \le \frac{a_u}{b_u} < \alpha_u$ apply. So, (x^*, y^*) satisfies (13).

ii. $\underline{B = \sum_{j \in N} \frac{a_j}{b_j}}$:

It will be shown that (15) is satisfied for some $\zeta \ge 0$. If $y^* = 0$, then (A) implies that $x_i^* \ge \frac{a_i}{b_i}$ for each $i \in N$. As $B \ge \sum_{j \in N} x_j^* \ge \sum_{j \in N} \frac{a_j}{b_j} = B$, it follows that $x_i^* = \frac{a_i}{b_i}$ for each $i \in N$. So, (15) holds with $\zeta = 0$. Next, assume that $y^* \ne 0$. We claim that $x_i^* \ge \frac{a_i}{b_i}$ for each $i \in N$. Indeed, if $x_i^* < \frac{a_i}{b_i}$ for some $i \in N$, then (C) implies that for some $w \in N$, $y_w^* > 0$ and $x_w^* < \frac{a_w}{b_w} \le \alpha_w$, in particular, $q_w(y_w^*) = c_w y_w^* > 0$. It now follows from Corollary 1 that $\sum_{j \in N} x_j^* = B = \sum_{j \in N} \frac{a_j}{b_j}$; as $x_w^* < \frac{a_w}{b_w}$, it further follows that $x_v^* > \frac{a_v}{b_v}$ for some $v \in N \setminus \{w\}$, implying that $p_v(x_v^*) = a_v - b_v x_v^* < 0$ and [by Lemma 1(i)] $y_v^* = 0$. But, $q_w(y_w^*) > 0 = c_v y_v^* = q_v(y_v^*)$, $x_v^* > \frac{a_v}{b_v} > 0$ and $x_w^* < \alpha_w$ contradict Corollary 1. The contradiction proves that $x_i^* \ge \frac{a_i}{b_i}$ for each $i \in N$. As $B \ge \sum_{j \in N} x_j^* \ge \sum_{j \in N} \frac{a_j}{b_j} = B$, it follows that $x_i^* = \frac{a_i}{b_i}$ for each $i \in N$, that is, x^* satisfies (15). In particular, $x_i^* = \frac{a_i}{b_i} = \alpha_i$ for each $i \in \nu_0$ and $x_i^* = \frac{a_i}{b_i} < \alpha_i$ for each $i \in \nu_-$. By Lemma 2, $q_i(y_i^*) = c_i y_i^*$ is a nonnegative constant for $i \in \nu_-$, say it equals ζ, and $q_i(y_i^*) = c_i y_i^* \ge \zeta$ for $i \in \nu_0$, so, y^* satisfies (15).

iii. $\underline{\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j} < B < \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} \text{ for some } k \in N}$:

It will be shown that (17) is satisfied with $\delta \equiv \frac{\sum_{j=1}^k \frac{a_j}{b_j} - B}{\sum_{j=1}^k \frac{1}{b_j}}$. Evidently, $\sum_{j \in N} \frac{a_j}{b_j} \ge \sum_{j=1}^k \frac{a_j - a_{k+1}}{b_j} > B$. Hence, (D) and (E) apply, assuring that (28) and (29) hold and for some $\alpha \in N$

$$\{1, \dots, \alpha\} = \{j \in N : y_j^* > 0\} \subseteq \arg\max_{j \in N} p_j(x_j^*), \tag{30}$$

[the inclusion following from Lemma 1(i) or Lemma 1(ii) and $y^* \ne 0$ following from (28)-(29)].

We will prove that $\alpha = k$. Let $\delta \equiv \max_{j \in N} p_j(x_j^*)$. By (30), (28), and (29), for $i \le \alpha$ and $u > \alpha$, $y_i^* > 0 = y_u^*$, $a_i - b_i x_i^* = p_i(x_i^*) = \delta \ge p_{\alpha+1}(x_{\alpha+1}^*) = a_{\alpha+1}$, $x_u^* = 0$, and $\sum_{j=1}^{\alpha} x_j^* = \sum_{j \in N} x_j^* = B$. Thus, $\frac{a_i - \delta}{b_i} = x_i^* \le \frac{a_i - a_{\alpha+1}}{b_i}$ for each $i \le \alpha$, and therefore $\sum_{j=1}^{\alpha} \frac{a_j - a_{\alpha+1}}{b_j} \ge \sum_{j=1}^{\alpha} x_j^* = B > \sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j}$. As $\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j}$ is strictly increasing in $k \le n-1$, it follows that $\alpha > k-1$, that is, $\alpha \ge k$. To establish a contradiction, assume that $\alpha \ge k+1$. Then for $i = 1, \dots, k$, $\delta = a_i - b_i x_i^* = p_i(x_i^*) = p_{k+1}(x_{k+1}^*) = a_{k+1} - b_{k+1} x_{k+1}^* \le a_{k+1}$, implying that $x_i^* \ge \frac{a_i - a_{k+1}}{b_i}$; so, $\sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} \le \sum_{j=1}^{k} x_j^* \le \sum_{j=1}^{\alpha} x_j^* = B < \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$, a contradiction.

As $\alpha = k$, we have that $x_i^* = \frac{a_i - \delta}{b_i}$ for $i \le k$ and $x_u^* = y_u^* = 0$ for $u > k$; so, x^* satisfies (17). Further, $\sum_{j=1}^{k} \frac{a_j - \delta}{b_j} = \sum_{j=1}^{k} x_j^* = \sum_{j \in N} x_j^* = B$, implying that $\delta = \frac{\sum_{j=1}^k \frac{a_j}{b_j} - B}{\sum_{j=1}^k \frac{1}{b_j}} > \frac{\sum_{j=1}^k \frac{a_j}{b_j} - \sum_{j=1}^k \frac{a_j - a_{k+1}}{b_j}}{\sum_{j=1}^k \frac{1}{b_j}} = a_{k+1} > 0$. Also, for $i = 1, \dots, k-1$, $a_i - b_i x_i^* = \delta = p_k(x_k^*) \le a_k$, implying that $x_i^* = \frac{a_i - \delta}{b_i} \ge \frac{a_i - a_k}{b_i} > 0$ [the last inequality follows from (10)]. Hence, $\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j} < B = \sum_{j \in N} x_j^* = x_k^* + \sum_{j=1}^{k-1} \frac{a_j - \delta}{b_j}$, assuring that $\delta < a_k$ and $x_k^* > 0$. It follows that $0 < x_i^* = \frac{a_i - \delta}{b_i} < \frac{a_i}{b_i} \le \alpha_i$ for $i \le k$ and $x_u^* = 0$ for $u > k$. So, Lemma 2 implies that for $i \le k$ $c_i y_i^* = q_i(y_i^*) = \eta \equiv \max_{j \in N} q_j(y_j^*)$, assuring that $y_i^* = \frac{\eta}{c_i}$. Further, as $x_1^* = \frac{a_1 - \delta}{b_1} < \frac{a_1}{b_1} \le \alpha_1$, (C) implies that $1 = \sum_{j \in N} y_j^* = \sum_{j=1}^{k} \frac{\eta}{c_j}$; hence, $\eta = \frac{1}{\sum_{j=1}^k \frac{1}{c_j}}$, completing the proof that y^* satisfies (17).

iv. $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j}$ for some $k \in \{1, \ldots, n-1\}$:

It will be shown that (19) is satisfied. Evidently, $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} < \sum_{j \in N} \frac{a_j}{b_j}$ and $B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} > \sum_{j=1}^{k} \frac{a_j - a_k}{b_j} = \sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j}$. As $B < \sum_{j \in N} \frac{a_j}{b_j}$, the arguments of case (iii) imply that for some $\alpha \in N$, (28)–(30) hold.

Again, let $\delta \equiv \max_{j \in N} p_j(x_j^*)$. Arguments used to prove part (ii), imply that $0 \le x_i^* = \frac{a_i - \delta}{b_i}$ for $i \le \alpha$, $x_u^* = 0$ for $u > \alpha$ and $\alpha \ge k$. As

$$\sum_{j=1}^{k} \frac{a_j - \delta}{b_j} \le \sum_{j=1}^{\alpha} \frac{a_j - \delta}{b_j} = \sum_{j=1}^{\alpha} x_j^* = B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j},$$

it follows that $a_{k+1} \le \delta$. Thus, for $j \ge k+2$, $p_j(x_j^*) \le a_j < a_{k+1} \le \delta$ and therefore, by (30) and (28), $x_j^* = y_j^* = 0$, in particular, $\alpha \le k+1$. So, $\alpha \in \{k, k+1\}$. As $a_j - a_{k+1} = 0$ for $j = k+1$, it follows that $\sum_{j=1}^{\alpha} \frac{a_j - \delta}{b_j} = B = \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} = \sum_{j=1}^{\alpha} \frac{a_j - a_{k+1}}{b_j}$. Thus, $\delta = a_{k+1} > 0$, $x_i^* = \frac{a_i - \delta}{b_i} = \frac{a_i - a_{k+1}}{b_i} > 0$ for $i \le k$ and $x_{k+1}^* = B - \sum_{j=1}^{k} x_j^* = B - \sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} = 0$. So, we established that x^* satisfies (19).

For $i \le k$, $0 < x_i^* = \frac{a_i - a_{k+1}}{b_i} < \frac{a_i}{b_i} \le \alpha_i$ and for $u > k+1$, $u > \alpha$ which implies $y_u^* = 0$ and [by (28)] $x_u^* = 0$. Further, $x_{k+1}^* = 0$. With $\theta \equiv \max_{j \in N}\{q_j(y_j^*) = c_j y_j^*\}$, it now follows from Lemma 2 that

$$c_i y_i^* = q_i(y_i^*) = \theta \ge q_{k+1}(y_{k+1}^*) = c_{k+1} y_{k+1}^* \quad \text{for } i \le k.$$

As $x_1^* = \frac{a_1 - a_{k+1}}{b_1} < \frac{a_1}{b_1}$, it follows that $p_1(x_1^*) = a_1 - b_1 x_1^* > 0$ and therefore, by Lemma 1(i), $1 = \sum_{j \in N} y_j^* = \sum_{j=1}^{k+1} y_j^*$. Consequently, $\eta \equiv y_{k+1}^* = 1 - \sum_{j=1}^{k} y_j^* = 1 - \sum_{j=1}^{k} \frac{\theta}{c_j}$, implying that $\theta = \frac{1-\eta}{\sum_{j=1}^{k} \frac{1}{c_j}}$ and for $i \le k$, $y_i^* = \frac{\theta}{c_i} = \frac{\frac{1-\eta}{c_i}}{\sum_{j=1}^{k} \frac{1}{c_j}}$. Finally, $\eta = y_{k+1}^* \ge 0$ is trite and

$$\left[\eta = y_{k+1}^* \le \frac{\theta}{c_{k+1}} = \frac{\frac{1-\eta}{c_{k+1}}}{\sum_{j=1}^{k} \frac{1}{c_j}}\right]$$
$$\Rightarrow \left[\eta\left(c_{k+1} + \frac{1}{\sum_{j=1}^{k} \frac{1}{c_j}}\right) \le \frac{1}{\sum_{j=1}^{k} \frac{1}{c_j}}\right]$$
$$\Rightarrow \left[\eta\left(\sum_{j=1}^{k} \frac{c_{k+1}}{c_j} + 1\right) \le 1\right]$$

Hence, y^* satisfies (19) with $0 \le \eta \le \frac{1}{\sum_{j=1}^{k+1} \frac{c_{k+1}}{c_j}}$. ||

APPENDIX C: PROOF OF THEOREM 2

Sufficiency

i. $B > \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$:

Assume that (x^*, y^*) satisfies (21) with corresponding ξ_i's and ρ_i's. Then, clearly, $x^* \in X$ and $y^* \in Y$. Also,

$$p_i(x_i^*) = \begin{cases} \tau & \text{if } i \in \widehat{\nu}_0 \\ a_i - b_i\xi_i \le \tau & \text{if } i \in \widehat{\nu}_- \\ a_i - b_i\xi_i \le a_i < \tau & \text{if } i \in N \setminus \widehat{N}, \end{cases}$$

implying that $\max_{j \in N} p_j(x_j^*) = \tau > 0$. A best response of the inspectee to x^* is then any vector $y \in Y$ satisfying $y \ge 0$, $\sum_{j \in N} y_i = 1$, and $y_i > 0$ only if $p_i(x_i^*) = \tau$, that is, $x_i^* = \frac{a_i - \tau}{b_i}$ [Lemma 1(i)]; in particular, y^* is such a response. On the other hand, if $y_i^* > 0$, then $i \in \widehat{\nu}_0$, implying that $x_i^* = \frac{a_i - \tau}{b_i} = \alpha_i$. So, $x_i^* = \alpha_i$ whenever $q_i(y_i^*) = c_i y_i^* > 0$. By Lemma 2, x^* is a best response of the inspector to y^*.

ii. $B = \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$:

Assume that $\zeta \ge 0$ and (x^*, y^*) satisfies (23) with corresponding ρ_i's . Then, clearly, $x^* \in X$ and $y^* \in Y$. Next, observe that $p_i(x_i^*) = a_i - b_i x_i^* = \tau > 0$ for each $i \in \widehat{N}$, and $p_i(x_i^*) = a_i < \tau$ for each $i \in N \setminus \widehat{N}$, implying [by Lemma 1(i)] that a best response of the inspectee to x^* is any vector $y \in Y$ satisfying $y_i \ge 0$ for $i \in \widehat{N}$, $y_i = 0$ for $i \in N \setminus \widehat{N}$, and $\sum_{j \in N} y_j = 1$. In particular, y^* is such a response. On the other hand, $\sum_{j \in N} y_j^* = 1$ assures that $\max_{j \in N}\{q_j(y_j^*) = c_j y_j^*\} > 0$ and

$$q_i(y_i^*) = c_i y_i^* = \begin{cases} c_i\rho_i \ge \zeta & \text{if } i \in \widehat{\nu}_0 \\ \zeta & \text{if } i \in \widehat{\nu}_- \setminus \{\mu\} \text{ or } i = \mu \text{ and } a_\mu > \tau \\ c_\mu\rho_\mu \le \zeta & \text{if } i = \mu \in \widehat{\nu}_- \text{ and } a_\mu = \tau \\ 0 & \text{if } i \in N \setminus \widehat{N}. \end{cases}$$

By Lemma 2, any vector $x \in X$ with $\sum_{i \in N} x_i = B$, $x_i = \alpha_i$ for each i with $q_i(y_i^*) > \zeta$, $0 \le x_i \le \alpha_i$ for each i with $q_i(y_i^*) = \zeta$, and $x_i = 0$ for each i with $q_i(y_i^*) < \zeta$ is a best response of the inspector to y^*. For the first of two cases assume that $a_\mu > \tau$. In this case, $q_i(y_i^*) > \zeta$ implies $i \in \widehat{\nu}_0$ for which $x_i^* = \frac{a_i - \tau}{b_i} = \alpha_i$, and $q_i(y_i^*) < \zeta$ implies $i \in N \setminus \widehat{N}$ for which $x_i^* = 0$; consequently x^* is a best response of the inspector to y^*. Alternatively, assume that $a_\mu = \tau$ (which implies $p_\mu(\alpha_\mu) < \tau$, that is, $\mu \in \nu_-$). In this case, $q_i(y_i^*) > \zeta$ implies $i \in \widehat{\nu}_0$ for which $x_i^* = \frac{a_i - \tau}{b_i} = \alpha_i$, further, $q_i(y_i^*) < \zeta$ implies $i \in N \setminus \widehat{N}$ or $i = \mu$ and in either case $x_i^* = 0$. So, again, x^* is a best response of the inspector to y^*.

iii. $0 < B < \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$:

Consider the model where each α_i is replaced by $\bar{\alpha}_i \equiv \max\{\alpha_i, \frac{a_i}{b_i}\} \ge \alpha_i$ while all other data elements remain unchanged. The model with the modified data will be referred to as the "bar" model (to be distinguished from the original model that we discuss). The strategy set of the inspector in the "bar" model is $\bar{X} \equiv \{x \in \mathbb{R}^n : 0 \le x_i \le \bar{\alpha}_i \text{ for } i \in N \text{ and } \sum_{j \in N} x_j \le B\} \supseteq X$. Also, for each $i \in N$, $p_i(\bar{\alpha}_i) = a_i - b_i\bar{\alpha}_i \le 0$, implying that $\{i \in N : \frac{a_i}{b_i} > \bar{\alpha}_i\} = \emptyset$ and consequently Theorem 1 applies to the "bar" model.

Assume that (x^*, y^*) is a Nash equilibrium of the "bar" model. As $0 < B < \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j} < \sum_{j \in N} \frac{a_j}{b_j}$, Theorem 1 implies that (x^*, y^*) satisfies either of the conditions given in (iii) or (iv) of that theorem, with a corresponding k (and these conditions do not depend on the $\bar{\alpha}_i$'s). As $\bar{X} \supseteq X$, to show that (x^*, y^*) is a Nash equilibrium it suffices to show that $x^* \in X$.

Assume first that (x^*, y^*) satisfies condition (iii) of Theorem 1 with $k \in N$, that is, (x^*, y^*) satisfies (17) with $\delta \equiv \frac{\sum_{j=1}^{k} \frac{a_j}{b_j} - B}{\sum_{j=1}^{k} \frac{1}{b_j}}$. To prove that $x^* \in X$ it suffices to show that $\frac{a_i - \delta}{b_i} \le \alpha_i$ for $i = 1, \ldots, k$. The definition of δ assures that $B = \sum_{j=1}^{k} \frac{a_j - \delta}{b_j}$ and the sufficiency proof of Theorem 1 assures that $a_{k+1} < \delta < a_k$. So,

$$\sum_{j=1}^{k-1} \frac{a_j - a_k}{b_j} = \sum_{j=1}^{k} \frac{a_j - a_k}{b_j} < \sum_{j=1}^{k} \frac{a_j - \delta}{b_j}$$
$$= B < \sum_{j=1}^{\mu} \frac{a_j - \tau}{b_j} < \sum_{j=1}^{\mu} \frac{a_j - a_{\mu+1}}{b_j} \quad (31)$$

which implies that $k - 1 < \mu$, that is, $k \le \mu$. If $k = \mu$, then (31) implies that $\delta > \tau$. Alternatively, if $k < \mu$, then $\delta > a_{k+1} \ge a_\mu \ge \tau$. In either case, for $i = 1, \dots, k$, $\frac{a_i - \delta}{b_i} < \frac{a_i - \tau}{b_i} \le \alpha_i$.

Next assume that (x^*, y^*) satisfies condition (iv) of Theorem 1 with $k \in \{1, \dots, n-1\}$, that is, (x^*, y^*) satisfies (19). To prove that $x^* \in X$, it suffices to show that $\frac{a_i - a_{k+1}}{b_i} \le \alpha_i$ for $i = 1, \dots, k$. Evidently, $\sum_{j=1}^{k} \frac{a_j - a_{k+1}}{b_j} = B < \sum_{j=1}^{\mu} \frac{a_j - \tau}{b_j} < \sum_{j=1}^{\mu} \frac{a_j - a_{\mu+1}}{b_j}$, implying that $k < \mu$, $a_{k+1} \ge a_\mu \ge \tau$, and for $i = 1, \dots, k$, $\frac{a_i - a_{k+1}}{b_i} \le \frac{a_i - \tau}{b_i} \le \alpha_i$.

Necessity

Suppose (x^*, y^*) is a Nash equilibrium and it will be shown that it satisfies the corresponding conditions. Recall that $\widehat{\nu}_0 \ne \emptyset$. Let $t \in \widehat{\nu}_0$. Then, $p_t(x_t^*) \ge p_t(\alpha_t) = \tau > 0$ and Lemma 1(i) implies that $\sum_{j \in N} y_j^* = 1$, in particular, $y^* \ne 0$. Further, observation (B) and the last conclusion of observation (C) of the proof of Theorem 1 are modified to:

- B' If $y_u^* > 0$ for $u \in \widehat{N}$, then (by Lemma 1(i)) $p_u(x_u^*) \ge \tau$, that is, $x_u^* \le \frac{a_u - \tau}{b_u}$.
- C' If $x_i^* < \frac{a_i - \tau}{b_i}$ for some $i \in N$ (that is, $p_i(x_i^*) > \tau$), then for some $w \in N$, $y_w^* > 0$ and $p_w(x_w^*) > \tau$, that is, $x_w^* < \frac{a_w - \tau}{b_w} \le \alpha_w$, in particular, $a_w \ge p_w(x_w^*) > \tau$, assures $w \in \widehat{N}$.

Observations (A)–(E) of the necessity proof of Theorem 1 will be used.

i. $\sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j} < B$:

It will be shown that (21) is satisfied with corresponding ξ_i's and ρ_i's. As $(x^*, y^*) \in X \times Y$ it suffices to show that $y_i^* = 0$ for $i \in (N \setminus \widehat{N}) \cup \widehat{\nu}_-$, $\sum_{j \in N} y_j^* = 1$ and $x_i^* \ge \frac{a_i - \tau}{b_i}$ for $i \in \widehat{N}$ (the latter would imply that for $i \in \widehat{\nu}_0$, $\frac{a_i - \tau}{b_i} \le x_i^* \le \alpha_i = \frac{a_i - \tau}{b_i}$ and therefore $x_i^* = \frac{a_i - \tau}{b_i}$). Consider $u \in N \setminus \widehat{N}$. As $p_u(x_u^*) \le a_u < \tau \le p_t(x_t^*)$, $u \notin \arg\max_{j \in N} p_j(x_j)$ and, by Lemma 1(i) or Lemma 1(ii), $y_u^* = 0$. Next, assume that $y_u^* > 0$ for some $u \in \widehat{\nu}_-$ and we will establish a contradiction. The assumption $y_u^* > 0$ implies that $q_u(y_u^*) = c_u y_u^* > 0$ and, by (B'), $x_u^* \le \frac{a_u - \tau}{b_u} < \alpha_u$. It now follows from Corollary 1 that $\sum_{j \in \widehat{N}} x_j^* = B > \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$ and therefore for some $v \in \widehat{N} \setminus \{u\}$, $x_v^* > \frac{a_v - \tau}{b_v} \ge 0$ and, by (B'), $y_v^* = 0$ or for some $j \in N \setminus \widehat{N}$, $x_j^* > 0$ and, as we showed, $y_j^* = 0$. But, $q_u(y_u^*) > 0 = c_v y_v^* = q_v(y_v^*)$ or $q_u(y_u^*) > 0 = c_j y_j^* = q_j(y_j^*)$, $x_v^* > 0$ or $x_j^* > 0$, and $x_u^* < \alpha_u$ contradict Corollary 1. Next, assume that $x_u^* < \frac{a_u - \tau}{b_u} \le \alpha_u$ for some $u \in \widehat{N}$ and we will establish another contradiction. As $x_u^* < \frac{a_u - \tau}{b_u}$, (C') implies that for some $w \in \widehat{N}$, $y_w^* > 0$ and $x_w^* < \frac{a_w - \tau}{b_w} \le \alpha_w$. Replacing u by w in the above arguments leading to a contradiction under the assumption that $y_u^* > 0$ and $x_u^* \le \frac{a_u}{b_u} < \alpha_u$ apply. So, (x^*, y^*) satisfies (21).

ii. $B = \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$:

It will be shown that for some $\zeta \ge 0$, (23) is satisfied with corresponding ρ_i's. We first prove that for each $i \in \widehat{N}$, $x_i^* \ge \frac{a_i - \tau}{b_i}$. Assume that this is not the case and for some $i \in \widehat{N}$, $x_i^* < \frac{a_i - \tau}{b_i}$ and we will establish a contradiction. By (C'), for some $w \in \widehat{N}$: $y_w^* > 0$, implying that $q_w(y_w^*) = c_w y_w^* > 0$, and $x_w^* < \frac{a_w - \tau}{b_w} \le \alpha_w$; thus, by Corollary 1, $\sum_{j \in N} x_j^* = B = \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$.

As $x_w^* < \frac{a_w - \tau}{b_w}$, it follows that for some $v \in N \setminus \{w\}$, either $v \in N \setminus \widehat{N}$ and $x_v^* > 0$, or $v \in \widehat{N}$ and $x_v^* > \frac{a_v - \tau}{b_v} > 0$, implying that $p_v(x_v^*) < \tau$. Hence, in either case $y_v^* = 0$ and $q_v(y_v^*) = c_v y_v^* = 0$. But, $q_w(y_w^*) = c_w y_w^* > 0 = q_v(y_v^*)$, $x_w^* < \alpha_w$ and $x_v^* > 0$ contradict Corollary 1. So, indeed, $x_i^* \ge \frac{a_i - \tau}{b_i}$ for each $i \in \widehat{N}$. Consequently, $B \ge \sum_{j \in N} x_j^* \ge \sum_{j \in \widehat{N}} x_j^* \ge \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j} = B$, implying that $x_i^* = 0$ for $j \in N \setminus \widehat{N}$ and $x_i^* = \frac{a_i - \tau}{b_i}$ for each $i \in \widehat{N}$, that is, x^* satisfies (23) (if $\mu \in \widehat{\nu}_-$ and $a_\mu = \tau$, then $\frac{a_\mu - \tau}{b_\mu} = 0$). It follows that $x_i^* = 0$ for $i \in N \setminus \widehat{N}$, $x_i^* = \frac{a_i - \tau}{b_i} = \alpha_i$ for each $i \in \widehat{\nu}_0$, $x_i^* = \frac{a_i - \tau}{b_i} < \alpha_i$ for each $i \in \widehat{\nu}_-$, $x_i^* > 0$ for $i \in \widehat{\nu}_- \setminus \{\mu\}$, $x_\mu^* > 0$ if $a_\mu > \tau$, otherwise $x_\mu^* = 0$. By Lemma 2 , $q_i(y_i^*) = c_i y_i^*$ is a nonnegative constant for $i \in \widehat{\nu}_- \setminus \{\mu\}$, say it equals ζ, further, $q_\mu(y_\mu^*) = c_\mu y_\mu^* = \zeta$ if $a_\mu > \tau$ and $q_\mu(y_\mu^*) = c_\mu y_\mu^* \le \zeta$ if $a_\mu = \tau$, and $q_i(y_i^*) = c_i y_i^* \ge \zeta$ for $i \in \widehat{\nu}_0$. Finally, for $i \in N \setminus \widehat{N}$, $p_i(x_i^*) \le a_i < \tau$ and (B') implies that $y_i^* = 0$. So, y^* satisfies (23).

iii. $0 < B < \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j}$:

Consider the "bar" model introduced under (iii) in the sufficiency proof with the corresponding definitions of the $\bar{\alpha}_i$'s and $\bar{X}$, and let (x^*, y^*) be a Nash equilibrium of the original model. To show that (x^*, y^*) is a Nash equilibrium of the "bar" model, it is sufficient to prove that for each $\bar{x} \in \bar{X} \setminus X$, $\widehat{U}^I(\bar{x}, y^*) \le \widehat{U}^I(x^*, y^*)$.

By (B') and by Lemma 1(i), if $y_i^* > 0$ for $i \in \widehat{N}$, then $p_i(x_i^*) \ge \tau$, that is, $x_i^* \le \frac{a_i - \tau}{b_i}$, and further, as $p_j(x_j^*) < \tau$ for $j \in N \setminus \widehat{N}$, $y_j^* = 0$ for $j \in N \setminus \widehat{N}$. Consider $\bar{x} \in \bar{X} \setminus X$. Then, there are indexes $v \in N$ such that $\frac{a_v}{b_v} > \alpha_v$, and $\bar{x}_v > \alpha_v \ge \frac{a_v - \tau}{b_v}$, implying that $p_v(\bar{x}_v) < \tau$, and so $y_v^* = 0$. As $0 < B < \sum_{j \in \widehat{N}} \frac{a_j - \tau}{b_j} < \sum_{j \in N} \frac{a_j - \tau}{b_j}$, there exists a $w \in \widehat{N}$, such that $\bar{x}_w < \frac{a_w - \tau}{b_w}$ and so $p_w(\bar{x}_w) > \tau$. By (C'), for some $r \in \widehat{N}$, $y_r^* > 0$ and $\bar{x}_r < \frac{a_r - \tau}{b_r}$. But, $\bar{x}_v > \frac{a_v - \tau}{b_v}$, $y_v^* = 0$, $\bar{x}_r < \frac{a_r - \tau}{b_r}$, and $y_r^* > 0$ contradict Corollary 1. In particular, $\bar{x} \in \bar{X} \setminus X$ cannot be a part of any Nash equilibrium, and so $\widehat{U}^I(\bar{x}, y^*) \le \widehat{U}^I(x^*, y^*)$.

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APPENDIX D: PROOF OF LEMMA 3

Part (i) is immediate from Theorem 1(i). As there are finitely many singular amounts which include $\sum_{j \in N} \frac{a_j}{b_j}$, the set of nonsingular amounts in $(0, \sum_{j \in N} \frac{a_j}{b_j}]$ is the union of disjoint open intervals to which Theorem 1(iii) applies. In particular, (17) shows that on any open interval of nonsingular amounts, each $y_i^*(\cdot)$ is the corresponding constant, proving (iii). Also, each $x_i^*(\cdot)$ is linear and weakly decreasing in δ. As δ is linear and decreasing in B, it follows that $x_i^*(\cdot)$ is linear and weakly increasing in B, further the slopes of x_i^* are available from (17) and are as stated in the lemma — as they are decreasing, and the concavity of $x_i^*(\cdot)$ on $[\sum_{j=1}^{i-1} \frac{a_j - a_i}{b_j}, \sum_{j=1}^{n} \frac{a_j}{b_j}]$ follows. To complete the proof of (ii), it remains to establish continuity of $x_i^*(\cdot)$ at singular values. For this purpose, express δ as a function of B [that is, write $\delta(B)$] and note that (17) assures that

$$\lim_{\epsilon\downarrow 0} \delta\left(\sum_{j=1}^{k}\frac{a_j - a_{k+1}}{b_j} - \epsilon\right) = \frac{\sum_{j=1}^{k}\frac{a_j}{b_j} - \left[\sum_{j=1}^{k}\frac{a_j - a_{k+1}}{b_j}\right]}{\sum_{j=1}^{k}\frac{1}{b_j}} = a_{k+1}$$

and

$$\lim_{\epsilon\downarrow 0} \delta\left(\sum_{j=1}^{k-1}\frac{a_j - a_k}{b_j} + \epsilon\right) = \frac{\sum_{j=1}^{k}\frac{a_j}{b_j} - \left[\sum_{j=1}^{k}\frac{a_j - a_k}{b_j}\right]}{\sum_{j=1}^{k}\frac{1}{b_j}} = a_k.$$

The continuity of each $x_i^*(\cdot)$ in B at singular amounts in now immediate from (17) and (19).

Finally, to prove (iv), observe from (19) of Theorem 1(iv) that for $k \in N \setminus \{n\}$ and singular amount $B = \sum_{j=1}^{k}\frac{a_j - a_{k+1}}{b_j}$, the range of $y_i^*(B)$ depends (linearly) on the parameter η. Using (19), for $i = 1, \ldots, k$, the extreme value $\eta = 0$ yields $\frac{\frac{1}{c_i}}{\sum_{j=1}^{k}\frac{1}{c_j}} = y_i^*(B-0)$ as the highest value of $y_i^*(B)$, and the alternative extreme value $\eta = \frac{1}{\sum_{j=1}^{k+1}\frac{c_{k+1}}{c_j}}$ yields

$$\frac{1 - \frac{1}{\sum_{j=1}^{k+1}\frac{c_{k+1}}{c_j}}}{c_i\sum_{j=1}^{k}\frac{1}{c_j}} = \frac{\sum_{j=1}^{k+1}\frac{c_{k+1}}{c_j} - 1}{(\sum_{j=1}^{k+1}\frac{c_{k+1}}{c_j})(c_i\sum_{j=1}^{k}\frac{1}{c_j})} = \frac{\sum_{j=1}^{k}\frac{c_{k+1}}{c_j}}{(\sum_{j=1}^{k+1}\frac{c_{k+1}}{c_j})(c_i\sum_{j=1}^{k}\frac{1}{c_j})} = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k+1}\frac{1}{c_j}} = y_i^*(B+0)$$

as the lowest value of $y_i^*(B)$; so, $y_i^*(B) = \left[y_i^*(B-0) = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k}\frac{1}{c_j}},\ y_i^*(B+0) = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k+1}\frac{1}{c_j}}\right]$. Further, again using (19), $0 = y_{k+1}^*(B-0)$ is the lowest value of $y_{k+1}^*(B)$ while $\frac{\frac{1}{c_{k+1}}}{\sum_{j=1}^{k}\frac{1}{c_j}}$ is the highest; so, $y_{k+1}^*(B) = [y_i^*(B-0) = 0,\ y_i^*(B+0) = \frac{\frac{1}{c_i}}{\sum_{j=1}^{k+1}\frac{1}{c_j}}]$. Finally, (19) assures that $y_i^*(B) = \{0\}$ for $i = k+2, \ldots, n$. ||

APPENDIX E: PROOF OF THEOREM 3

By Lemma 3(i), $y^*(B) = \{y \in Y(B) : y_i = 0 \text{ for each } i \in \nu_- \text{ and } y_i \geq 0 \text{ for each } i \in \nu_0\}$ for $B > \sum_{j\in N}\frac{a_j}{b_j}$ and therefore (9), (4) assure that $\widehat{U}^I(x,y)$ and $U^V(x,y)$ given by (14) can now be rewritten as $(\widehat{U}^I)^*(B) \geq 0$ and $(U^V)^*(B) = 0$.

Consider $k \in N$ and $B \in I_k$. By Lemma 3, $(U^V)^*(B) = \sum_{i=1}^{k}\left[a_i - b_i x_i^*(B)\right]y_i^*(B)$ and $(\widehat{U}^I)^*(B) = \sum_{i=1}^{k} c_i x_i^*(B) y_i^*(B)$ with each $x_i^*(\cdot)$ linear and $y_i^*(\cdot)$ constant on I_k, implying that $(U^V)^*(\cdot)$ and $(\widehat{U}^I)^*(\cdot)$ are linear on I_k. The explicit expressions for the slopes of the the x_i^*'s and the value of the y_i^*'s on I_k imply that the slopes of $(U^V)^*(\cdot)$ and $(\widehat{U}^I)^*(\cdot)$ are, respectively,

$$\sum_{i=1}^{k}(-b_i)\left(\frac{\frac{1}{b_i}}{\sum_{j=1}^{k}\frac{1}{b_j}}\right)\left(\frac{\frac{1}{c_i}}{\sum_{j=1}^{k}\frac{1}{c_j}}\right) = \left(\frac{-1}{\sum_{j=1}^{k}\frac{1}{b_j}}\right)\left(\frac{\sum_{i=1}^{k}\frac{1}{c_i}}{\sum_{j=1}^{k}\frac{1}{c_j}}\right) = \frac{-1}{\sum_{j=1}^{k}\frac{1}{b_j}};$$

and

$$\sum_{i=1}^{k}c_i\left(\frac{\frac{1}{b_i}}{\sum_{j=1}^{k}\frac{1}{b_j}}\right)\left(\frac{\frac{1}{c_i}}{\sum_{j=1}^{k}\frac{1}{c_j}}\right) = \left(\frac{\sum_{i=1}^{k}\frac{1}{b_i}}{\sum_{j=1}^{k}\frac{1}{b_j}}\right)\left(\frac{1}{\sum_{j=1}^{k}\frac{1}{c_j}}\right) = \frac{1}{\sum_{j=1}^{k}\frac{1}{c_j}};$$

the first set of constants are negative and increasing in k whereas the second set is positive and decreasing in k. This completes the proof of (iii). To complete the proof of (ii), it remains to show that for any singular amount B, $(U^V)^*(B)$ is a singleton and that $(U^V)^*(\cdot)$ is continuous at B. So, let B be a singular amount. Lemma 3(ii) assures that $x^*(B)$ is a singleton [to be denoted $x^*(B)$] and therefore for all $y^* \in y^*(B)$, $(U^V)(x^*(B), y^*)$ equals $\max_{y\in Y}(U^V)(x^*(B), y)$ which is independent of y^*. So, $(U^V)^*(B)$ contains a single value. Further, Lemma 3(iv) assures that this value equals the left and right limits of $(U^V)^*(\cdot)$ at B, completing the proof of (ii).

Finally, to prove (iv) consider singular amount $B = \sum_{j=1}^{k}\frac{a_j - a_{k+1}}{b_j}$ where $k \in N$. It then follows from (9) and parts (ii) and (iv) of Lemma 3, that (20) can be written as

$$(\widehat{U}^I)^*(B) = \left[\sum_{j=1}^{k}c_j\left(\frac{a_j - a_{k+1}}{b_j}\right)\left(\frac{\frac{1}{c_j}}{\sum_{u=1}^{k}\frac{1}{c_u}}\right),\ \times\sum_{j=1}^{k}c_j\left(\frac{a_j - a_{k+1}}{b_j}\right)\left(\frac{\frac{1}{c_j}}{\sum_{u=1}^{k+1}\frac{1}{c_u}}\right)\right] \quad (32)$$

$$= \left[\frac{\sum_{j=1}^{k}\left(\frac{a_j - a_{k+1}}{b_j}\right)}{\sum_{j=1}^{k}\frac{1}{c_j}},\ \frac{\sum_{j=1}^{k}\left(\frac{a_j - a_{k+1}}{b_j}\right)}{\sum_{j=1}^{k+1}\frac{1}{c_j}}\right], \quad (33)$$

$(\widehat{U}^I)^*(B-0) = \frac{\sum_{j=1}^{k}\left(\frac{a_j - a_{k+1}}{b_j}\right)}{\sum_{j=1}^{k}\frac{1}{c_j}}$ and $(\widehat{U}^I)^*(B+0) = \frac{\sum_{j=1}^{k}\left(\frac{a_j - a_{k+1}}{b_j}\right)}{\sum_{j=1}^{k+1}\frac{1}{c_j}}$. ||

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