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# Sustainable infrastructure investment with labor-only production

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**Abstract**

The purpose of this paper is to consider a partial equilibrium model for a sustainable infrastructure investment in a labor-production economy. We consider an inter-temporal Stackelberg game in a ‘‘capital primitive’’ economy where all capital investments are made by a Central Agency (a government). The government is assumed to have a number of objectives including sustainability of the infrastructure investments, while firms are assumed to be myopic, maximizing only current profits and paying taxes as a function of their returns. Both open-loop and closed-loop (feedback) Stackelberg strategies are considered. Based on the analysis of the investment game, some conclusions are drawn regarding the propensity to invest as a function of sustainability constraints, the taxation rates and employment levels. We then show that investments can tend to a constant level and thus strategic government goals of sustainability and employment growth can be planned only if labor costs and the general price index are steady or characterized by a set of conditions ensuring the attainability of the steady-state investment.


*Keywords:* Investment; Infrastructure; Stackelberg equilibrium

## 1. Introduction

The economic effects of infrastructure investments have been a topic of intense study (for example, see Aschauer, 1989a, b; Holtz-Eakin and Schwartz, 1994, 1995; Munnel, 1990a, b, 1991; Anwar, 2001). In seminal papers, Aschauer (1989a, b) pointed out that infrastructure investment is underestimated, neglecting the productivity of public capital. Subsequent macro-econometric studies have reinforced Aschauer’s proposition, pointing out to an appreciable and positive elasticity between infrastructure investments and GNP growth. This effect was explained by the appreciable economic contribution of highway construction (and other logistic infrastructure), which lowers transportation costs, increases the time value for individual drivers, etc. (Batten and Karlsson, 1996; Berndt and Harisson, 1991; Nijkamp and Blaas, 1993). Isolating specific investments, such as a particular investment in a highway, in augmenting production and its efficiency or in a logistic hub was more difficult to

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determine, however, and, therefore, investment in projects infrastructure is more difficult to justify (see also Gramlich, 1994; Morrison and Schwartz, 1992, 1996; Manning et al., 1985; Evans and Karras, 1994; Cain, 1997).

In a paper by Sanchez-Robles (1998a, b), a broad number of analyses have shown a wide dispersion but still high levels of elasticity. Munnell (1992) assessing a number of studies concluded that infrastructure research is still not conclusive for policy making in individual projects. Cain (1997) while providing a perspective on US industrial development, stresses the link between infrastructure investment and the US economic development, and concludes that infrastructure investment produces both direct and indirect effects. Cliometricians' estimates of the direct effects suggest that the initial public expenditures were warranted; there is a link between public infrastructure investment and private economic growth. Morrison and Schwartz (1996) provide a case study on the economic performance impacts of public infrastructure investment. These impacts are explored by evaluating substitution patterns affecting private input use in New England manufacturing. Using a cost-based methodology, it is found that in the short run, public capital expenditures provide cost-saving benefits that exceed the associated investment costs due to substitutability between public capital and private inputs. Over time, however, they find that stimulating investment in private capital increases economic performance more effectively than public capital expenditures alone and in fact reduces the cost incentive for such expenditures. In addition, growth in output motivated by infrastructure investment increases employment opportunities because this growth overrides short-run substitutability. Similarly, while assessing the effects of "shared resources", Corbey (1991) indicates that "infrastructure" (my italics) investments in flexible capacities that tend to be large are a precondition to a solid economic infrastructure for production efficiency. By the same token, Lu and Yang (2007) refer to the effects of investments in logistics zones that are shared logistics systems (see also Parola and Sciomachen (2004) for a transport and simulation-related problem). And finally, Manecke and Schoensleben (2004), (see also Akkermans and van der Horst, 2002) address the "infrastructure" issues associated with investments.

Additional and numerous references on this topic can be found including for example, Nagaraj et al. (1999), Rioja (1999), Binder and Smith (1996), Jansson (1993) as well as many other papers dealing with the specific effects of public investments. As a result, both theoretical and empirical research to assess the effects of infrastructure investment is needed (for additional related papers see, for example, Bradford, 1975; Ford and Poret, 1991; Gillen, 1996; Shah, 1992; Kwan et al., 1999 as well as related issues in Kogan and Tapiero, 2007).

These studies, while emphasizing the importance of this topic to better understand the role of government (and collective economic entities) assume that markets are competitive and do not consider the role of public institutions in providing investments and support for infrastructure. This is particularly the case in capital-poor countries as well as in logistics infrastructure investments such as Ports, Highways, Bridges, IT networks, etc. In such contexts, a leader-led relationship between investors (say a public authority or a collective entity) and industrial firms which require such an infrastructure to both employ people and profit (and thereby pay taxes to the public authority) is needed. In this paper, we introduce such a framework based on a Stackelberg differential game and provide a number of insights for infrastructure investment in a capital-poor environment where individual firms may not be able to invest into the infrastructure but require such an infrastructure for their economic activities. Examples to this effect abound. The financial banking infrastructure offered by the World Bank and local banks to provide "pennies" financing to entrepreneurs devoid of any means is such an outstanding example. By the same token, in a "capital primitive" economy, infrastructure investments for production are supplied by the government and in some cases by international economic agencies or foreign governments. In such cases, employment growth rate targets can be met if both taxation rates and capital investments are sustainable. Similar investments (although on a much smaller scale) are made by some supply chains who invest in IT and in other means to render operations within the supply chain more efficient.

The purpose of this paper is to consider an infrastructure investment for production in a capital-poor economy. In particular, in order to maintain our analysis tractable and useful, we make a number of simplifying assumptions allowing us to obtain theoretical and analytical results, providing some insights regarding the investment process in such an environment. Explicitly, our analysis will use an inter-temporal model for an economy where all capital investments are made by a public authority (such as a government), acting as a leader in the Stackelberg game while production firms (say individual entrepreneurs) are followers. For simplicity, we assume that firms are represented by a synthetic firm who requires a minimal sustainable profit and who uses the infrastructure and available labor to optimize profits. These profits are then taxed, generating thereby revenues for the government who in turn invests such revenues (partly or wholly) to replace and augment the infrastructure capital. While the government assumes a number of objectives, the firm is assumed to be myopic only maximizing its profits. Such a reasonable assumption presumes, of course, that the public authority is long-sighted while individual entrepreneurs (firms) are short-sighted. Our analysis considers then both open-loop and feedback Stackelberg strategies and provides conditions to allow sustainable rates for both investments and employment. Additional and practical implications are discussed as well in the paper, while mathematical developments are relegated to the Appendix.

## 2. A dynamic model

Let the aggregate production function expressing the amounts of goods produced by a synthetic firm be $Q=f(G,K,L)$, a function of the (publicly available) capital infrastructure $K$, a private capital $G$ owned by the firm and $L$, the labor employed by this firm. If the synthetic firm has no private capital and uses only the capital infrastructure and labor, then the production function is given by $Q=f(K,L)$. Let the general price index for the firm output be $p(t)$ at time $t$ and $c_{\mathrm{L}}(t)$ be the unit labor wage, both assumed known at time $t$. Further, let $\tau$ be a constant taxation rate. A firm's gross profit at time $t$ is then $P(t)=p(t)f(K(t),L(t))-c_{\mathrm{L}}(t)L(t)$, while its net profit is $(1-\tau)P(t)$. Let $\pi$ be the required profit and sustainable profit of the synthetic firm or $(1-\tau)P(t)\geqslant\pi>0$. Note that such a profit is necessarily positive, for otherwise, there will be no incentive for the firm to operate. In this case, the taxation rate is necessarily bounded by $\tau\leqslant 1-\pi/P(t)$. Let $K(t)$ be the level of current infrastructure capital, deteriorating at the rate $\delta$ and let $I(t)$ be the level of infrastructure investment. Thus,

$$\frac{\mathrm{d}K(t)}{\mathrm{d}t}=-\delta K(t)+I(t),\quad K(0)=K_0,\quad I(t)\geqslant 0. \quad (1)$$

Government revenues from the synthetic firm economic activities are given by $\tau P(t)$, while investment costs are assumed to be a convex function $C_{\mathrm{I}}(I)$, $C_{\mathrm{I}}'(I)>0$ for $I\neq 0$ and $C_{\mathrm{I}}''(I)>0$. In case of no borrowing, revenues, net of investment costs, are non-negative and then given by

$$\tau P(t)-C_{\mathrm{I}}(I(t))\geqslant 0 \quad (2)$$

and, therefore, the taxation constraint is

$$\frac{C_{\mathrm{I}}(I(t))}{P(t)}\leqslant\tau\leqslant 1-\pi/P(t). \quad (3)$$

The government and the synthetic firm problem consists then in an investment game $I(t)\geqslant 0$ with a fixed taxation rate, $\tau\geqslant 0$, to sustain revenues, and, of course, to sustain the synthetic firm's profitability, while the firm—the follower—determines the employment level that maximizes its profits. This situation is given by the following objectives:

$$\underset{I(t)\geqslant 0,\tau\geqslant 0}{\mathrm{Max}}\int_0^\infty \mathrm{e}^{-rt}(\tau P(t)-C_{\mathrm{I}}(I(t)))\,\mathrm{d}t,$$
$$\underset{L(t)\geqslant 0}{\mathrm{Max}}(1-\tau)P(t), \quad (4)$$

which are subject to constraints (1)–(3). As stated earlier, this is a Stackelberg game that we consider below for both the open-loop and feedback strategies. The solution of this problem (albeit as a partial equilibrium model) may provide some results of practical usefulness to infrastructure policy makers.

## 3. The Stackelberg investment and labor game

In our Stackelberg game, the public authority acts as a leader, fully abreast of the synthetic firm objective and constraints, while the firm is a follower, optimizing its profits subject to the taxation rate and the capital infrastructure. For simplicity, let the aggregate production function $Q=f(K,L)$ be of the Cobb–Douglas type, $f(K,L)=aK^{\alpha}L^{\beta}$. Further, assume that the firm's optimal employment level has an interior solution, meeting the firm's survivability constraint (3) and, specifically,

$$(1-\tau)P(t)\geqslant\pi. \quad (5)$$

Such an assumption is reasonable since in a Stackelberg game the leader will lead the follower to pay as much taxes as possible, while at the same time he will seek to have the follower meet its sustainability constraint. In this case, the follower (the synthetic firm) chooses an employment level that maximizes his profits, while the public institution will determine a taxation rate that will generate as much income as possible and at the same time sustain the synthetic firm activity. In this case, the firm's optimal decision to employ labor is given by $((1-\tau)\partial P/\partial L = 0)$

$$L^*(t) = K^{\alpha/(1-\beta)}(t)\left[\frac{p(t)a\beta}{c_L(t)}\right]^{1/(1-\beta)}$$

or

$$L^*(t) = \xi(t)K^{\alpha/(1-\beta)}(t), \tag{6}$$

where

$$\xi(t) = \left[\frac{p(t)a\beta}{c_L(t)}\right]^{1/(1-\beta)}. \tag{7}$$

However, when the profit constraint is binding, the condition for optimal profits is

$$p(t)aK^{\alpha}(t)L^{*\beta}(t) - c_L(t)L^*(t) = \pi^*/(1-\tau). \tag{8}$$

Then the optimal employment level $L^*(t)$ is found from (8) rather than (6). Since this does not affect the approach, we henceforth focus on an interior solution. For an interior solution, a firm's profit is given by (after some elementary manipulations)

$$P(t) = K^{\alpha/(1-\beta)}(t)\varphi(t)\left[\frac{1}{\beta}-1\right],$$

$$\varphi(t) = \xi(t)c_L(t) = c_L(t)\left[\frac{p(t)a\beta}{c_L(t)}\right]^{1/(1-\beta)}, \tag{9}$$

which determines as well the interior (and least profit) survivability constraint or

$$P(t) = K^{\alpha/(1-\beta)}(t)\varphi(t)\left[\frac{1}{\beta}-1\right] \geqslant \frac{\pi}{1-\tau}. \tag{10}$$

Assuming as stated earlier that there is no borrowing, $\tau P(t) - C_I \geqslant 0$, then using the Cobb–Douglas function and the interior firm's solution, the government Stackelberg policy is given by the following optimal control problem:

$$\underset{I(t)\geqslant 0,\tau\geqslant 0}{\mathrm{Max}} \int_0^{\infty} \mathrm{e}^{-rt}\left(\tau K^{\alpha/(1-\beta)}(t)\varphi(t)\left\{\frac{1}{\beta}-1\right\} - C_I(I(t))\right)\mathrm{d}t. \tag{11}$$

Subject to (1) and to the sustainability constraint

$$C_I(I(t)) \leqslant \tau K^{\alpha/(1-\beta)}(t)\varphi(t)\left\{\frac{1}{\beta}-1\right\}. \tag{11a}$$

A solution of this problem, a standard control optimization problem, will indicate the government policies in taxation and infrastructure investments in a labor-only economy. Furthermore, it is easy to verify that if $\alpha+\beta \leqslant 1$, then the objective function (11) is a sum of concave functions and constraints (1) and (11a) form a convex set. Therefore, problem (1), (11a), (11) is a concave program and any local optimum that we find will be a global one. These observations underpin the open- and closed-loop solutions we shall derive below.

## 4. The open-loop equilibrium analysis

Assuming an optimal employment (Eq. (6)), and given an optimal $I^*(t)$ and $K^*(t)$, note that an optimal tax rate $\tau^*$ is set such that the sustainability condition (10) holds for any $t$:

$$\underset{t\geqslant 0}{\mathrm{Min}}[p(t)f(K^*(t), L^*(t)) - c_L(t)L^*(t)] = \frac{\pi}{1-\tau^*}. \tag{12}$$

For the Cobb–Douglas production function, it is reduced to

$$\underset{t\geqslant 0}{\mathrm{Min}}\, K^{\alpha/(1-\beta)}(t)\varphi(t)\left\{\frac{1}{\beta}-1\right\} = \frac{\pi}{1-\tau^*}. \tag{12a}$$

Thus, we can first solve (11), (11a) for an arbitrary taxation rate, and subsequently determine the optimal tax rate. Namely, let

$$\Gamma(t) = C_I(I(t)) - \tau K^{\alpha/(1-\beta)}(t)\varphi(t)\left\{\frac{1}{\beta}-1\right\}, \tag{13}$$

and consider the Hamiltonian:

$$H(t) = \mathrm{e}^{-rt}\left(\tau K^{\alpha/(1-\beta)}(t)\varphi(t)\left\{\frac{1}{\beta}-1\right\} - C_I(I(t))\right) + \psi(t)(I(t) - \delta K(t)), \tag{14}$$

where the co-state variable $\psi(t)$ (the marginal cost of capital accumulation) is defined by the co-state equation:

$$\dot{\psi}(t) = -\frac{\partial H}{\partial K} + \lambda(t)\frac{\partial \Gamma}{\partial K}. \tag{14a}$$

Using (13) and (14), we have

$$\dot{\psi}(t) = -\left(\tau\frac{\alpha}{\beta}K^{(\alpha/(1-\beta))-1}(t)\varphi(t)\right)\left(\mathrm{e}^{-rt} + \lambda(t)\right) + \delta\psi(t) \tag{14b}$$

$$\lim_{t\to\infty}\psi(t) = 0. \tag{14c}$$

The non-negative multiplier $\lambda(t)$ is due to the mixed constraint (11a) and is found from the local maximum principle and the complementary slackness condition:

$$\frac{\partial H}{\partial I} - \lambda(t)\frac{\partial \Gamma}{\partial I} = 0, \quad \lambda(t)\Gamma(t) = 0. \tag{15}$$

Thus,

$$\lambda(t) = \frac{\psi(t)}{\partial C_{\mathrm{I}}(t)/\partial I(t)} - \mathrm{e}^{-rt}$$
$$\text{if } C_{\mathrm{I}}(I(t)) = \tau K^{\alpha/(1-\beta)}(t)\varphi(t)[\beta^{-1} - 1], \quad (16)$$

$$\lambda(t) = 0 \quad \text{if} \quad C_{\mathrm{I}}(I(t)) < \tau K^{\alpha/(1-\beta)}(t)\varphi(t)[\beta^{-1} - 1]. \quad (17)$$

According to the maximum principle, the Hamiltonian is maximized as a function of its admissible controls. Thereby considering only $I(t)$-dependent terms of the Hamiltonian, we have

$$\underset{I(t)\geqslant 0}{\mathrm{Max}}\, H(I(t)) = -C_{\mathrm{I}}(I(t))\mathrm{e}^{-rt} + \psi(t)I(t), \quad (18)$$

which provides the following optimality condition:

$$I(t) = \begin{cases} \gamma & \text{if } \psi(t) \geqslant \frac{\partial C_{\mathrm{I}}(0)}{\partial I}\mathrm{e}^{-rt}, \\ 0 & \text{otherwise}, \end{cases} \quad (19)$$

where the Stackelberg investment strategy, $\gamma = F(\psi(t))$, is defined by

$$\psi(t) = \frac{\partial C_{\mathrm{I}}(\gamma)}{\partial I}\mathrm{e}^{-rt}. \quad (20)$$

Explicit solutions are considered next with proofs introduced for clarity in a Mathematical Appendix. To simplify the analysis, we shall assume that $\partial C_{\mathrm{I}}(0)/\partial I = 0$. A first observation relates to a steady-state solution, which is characterized by a constant level of investment. In other words, over the long run, a public authority investing in infrastructure and collecting taxes can (in the conditions and the limitations explicitly stated by our problem) use a fixed investment policy. Such results presume as well a stable ratio of prices to labor costs such that $\varphi(t) = \bar{\varphi}$.

**Lemma 1.** *Consider the differential game* (1)–(4). *Let* $\varphi(t) = \bar{\varphi}$ *for* $t \geqslant 0$ *and* $\bar{K}$ *be defined by*

$$\frac{\partial C_{\mathrm{I}}(\delta\bar{K})}{\partial I}(r+\delta) = \tau\frac{\alpha}{\beta}\bar{K}^{(\alpha+\beta-1)/(1-\beta)}\bar{\varphi}.$$

*If* $K_0 = \bar{K}$ *and* $C_{\mathrm{I}}(\delta\bar{K}) \leqslant \tau\bar{K}^{\alpha/(1-\beta)}\bar{\varphi}[\beta^{-1} - 1]$, *then there is a steady-state Stackelberg equilibrium*, *which is* $K^*(t) = \bar{K}$ *and* $I^*(t) = \delta\bar{K}$ *for* $t \geqslant 0$.

To study the effect of the production elasticity $\alpha$ with respect to capital infrastructure on the optimal steady-state investment, we apply implicit differentiation to the infrastructure capital equation $R = \left(\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I\right)(r+\delta) - \tau(\alpha/\beta)\bar{K}^{(\alpha/(1-\beta))-1}\bar{\varphi} = 0$. This results in the equation

$$\frac{\mathrm{d}\bar{K}}{\mathrm{d}\alpha} = \frac{\left(1 + (\alpha/(1-\beta))\left((\alpha/(1-\beta)) - 1\right)\bar{K}^{-1}\right)\bar{K}^{(\alpha/(1-\beta))-1}\tau/\beta\bar{\varphi}}{\left(\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I^2\right)\delta(r+\delta) - \left((\alpha/(1-\beta)) - 1\right)\bar{K}^{(\alpha/(1-\beta))-2}\tau/\beta\bar{\varphi}}.$$

Thus we straightforwardly conclude with the following propositions.

**Proposition 1.** *If the production elasticity with respect to the infrastructure capital*, $\alpha$, *is such that* $\left(\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I^2\right)\delta(r+\delta) - \alpha\left(\alpha/(1-\beta) - 1\right)$ $\bar{K}^{(\alpha/(1-\beta))-2}$ $(\tau\bar{\varphi}/\beta) > 0$ *and the production elasticity with respect to the labor* $\beta$ *is such that* $1 + \left(\alpha/(1-\beta)\right)\left(\alpha/(1-\beta) - 1\right)\bar{K}^{-1} > 0$, *then the lower the elasticity with respect to the infrastructure*, *the lower the steady-state investment and infrastructure capital*.

It is easy to observe that the conditions for Proposition 1 always hold for small capital elasticity $\alpha$, if the production elasticity with respect to the labor, $\beta$, is low as well. On the other hand, if the production elasticity with respect to the labor is close to 1, the steady-state capital is inversely proportional to $\alpha$ regardless of the magnitude of $\alpha$, as stated in the following proposition.

**Proposition 2.** *If the production elasticity with respect to the labor*, $\beta$, *is close to* 1, *then the lower the elasticity with respect to the infrastructure*, *the higher the steady-state investment and infrastructure capital*.

Naturally, if the initial capital, $K_0$, differs from the steady-state capital, $\bar{K}$, then the optimal investment strategy is time variant. Of course, price and labor-costs inflations will lead also to non-stationary policies. The following lemma shows when the steady state is attractive, an equilibrium investments will tend to attain it.

**Lemma 2.** *Consider the differential game* (1)–(4). *If* $K_0 \neq \bar{K}$, $\varphi(t) = \bar{\varphi}$ *for* $t \geqslant 0$ *and* $C_{\mathrm{I}}(F(d(t))) \leqslant \tau b^{\alpha/(1-\beta)}(t)\bar{\varphi}(\beta^{-1} - 1)$, *where* $b(t)$ *and* $d(t)$ *satisfy System A* (*see Appendix*), *then the open-loop Stackelberg equilibrium is* $K^*(t) = b(t)$ *and* $I^*(t) = F(d(t))$ *for* $t \geqslant 0$, $\lim_{t\to\infty} K^*(t) = \bar{K}$ *and* $\lim_{t\to\infty} I^*(t) = \delta\bar{K}$.

To highlight these results, we shall consider some simple examples providing explicit results and allowing evident interpretations.

**Example 1.** Let $C_{\mathrm{I}}(I) = c_{\mathrm{I}}I^2$, then $\bar{K} = [2c_{\mathrm{I}}\delta(r+\delta)\beta/\tau\alpha\bar{\varphi}]^{(1-\beta)/(\alpha+2\beta-2)}$. Next, let $\alpha + \beta = 1$, then $\bar{K} = \left[2c_{\mathrm{I}}\delta(r+\delta)\beta/\tau\alpha\bar{\varphi}\right]^{-1}$, and from Lemma 2,

$\dot{d}(t) = -\mathrm{e}^{-rt}\tau\alpha\bar{\varphi}/\beta + \delta d(t)$. The solution of this differential equation is $d(t) = (\mathrm{e}^{-rt}/(r+\delta)(\tau\alpha/\beta)\,\bar{\varphi} + A\mathrm{e}^{\delta t} = 2c_{\mathrm{I}}\delta\bar{K}\mathrm{e}^{-rt} + A\mathrm{e}^{\delta t}$. With respect to the boundary condition for $d(t)$, $\lim_{t\to\infty} d(t) = 0$, we have $A = 0$. Substituting this solution as well as Eq. (20) into (11a) we find

$$\dot{b}(t) = -\delta b(t) + \frac{1}{2c_{\mathrm{I}}} d(t)\mathrm{e}^{rt} = -\delta b(t) + \delta\bar{K}.$$

The solution of this differential equation is evidently given by $b(t) = \bar{K} + B\mathrm{e}^{-\delta t}$. Taking into account that $b(0) = K_0$, we have $B = K_0 - \bar{K}$. Accordingly, we have determined that

$$K^*(t) = \bar{K} + (K_0 - \bar{K})\mathrm{e}^{-\delta t}, \quad I^*(t) = \frac{d(t)}{2c_{\mathrm{I}}}\mathrm{e}^{rt} = \delta\bar{K}.$$

Thus, the equilibrium capital asymptotically tends to the steady-state level $\bar{K}$ from above, when $K_0 > \bar{K}$, and from below, if $K_0 < \bar{K}$. Furthermore, if $\alpha + \beta = 1$ and $C_{\mathrm{I}}(I) = c_{\mathrm{I}}I^2$, there exists a constant investment level, $I(t) = \delta\bar{K}$ for $t \geqslant 0$, which ensures that the equilibrium infrastructure capital attains the steady state.

Consequently, we may assume that a government investment policy is sustainable at least at the steady state, i.e., the condition, $C_{\mathrm{I}}(\delta\bar{K}) \leqslant \tau\bar{K}^{\alpha/(1-\beta)}\bar{\varphi}[\beta^{-1} - 1]$ of Lemma 1, holds. This means with respect to our example that $c_{\mathrm{I}}\delta^2\bar{K}^2 \leqslant \tau\bar{K}^{\alpha/(1-\beta)}\bar{\varphi}(\beta^{-1} - 1)$, where $c_{\mathrm{I}}$ is the marginal investment cost. We observe also that the sustainability condition of Lemma 2, $C_{\mathrm{I}}(F(d(t))) \leqslant \tau b^{\alpha/(1-\beta)}(t)\bar{\varphi}(\beta^{-1} - 1)$ takes now the following form $c_{\mathrm{I}}\delta^2\bar{K}^2 \leqslant \tau b^{\alpha/(1-\beta)}(t)\bar{\varphi}(\beta^{-1} - 1)$, and always holds, if $K_0 > \bar{K}$. That is, government investments are always profitable if the initial infrastructure capital exceeds sustainable, steady-state capital. On the other hand, if $K_0 < \bar{K}$, then we obtain the following sufficient condition for the entire planning horizon sustainability condition:

$$c_{\mathrm{I}}\delta^2\bar{K}^2 \leqslant \tau K_0^{\alpha/(1-\beta)}(t)\bar{\varphi}(\beta^{-1} - 1).$$

Substituting the value for the steady-state capital, we have

$$c_{\mathrm{I}}\delta^2\left[\frac{2c_{\mathrm{I}}\delta(r+\delta)\beta}{\tau\alpha\tau K_0^{\alpha/(1-\beta)}\bar{\varphi}}\right]^{-2} \leqslant \tau K_0^{\alpha/(1-\beta)}\bar{\varphi}(\beta^{-1} - 1).$$

Simple manipulations indicate a lower bound on initial capital $K_0$, which along with the condition $K_0 < \bar{K}$ shows the sensitivity of the equilibrium investments to the sustainability constraint:

$$K_0 \geqslant \frac{\tau\omega\alpha}{4c_{\mathrm{I}}(r+\delta)^2\beta}[a\beta]^{1/\alpha} = \hat{K},$$

$$K_0 < \bar{K} = \frac{\tau\omega\alpha}{2c_{\mathrm{I}}\delta(r+\delta)\beta}[a\beta]^{1/\alpha}.$$

Namely, the higher the discount and infrastructure deterioration rates, the wider the range $\left[\hat{K}, \bar{K}\right]$, the less binding the sustainability constraint and thereby the less initial capital is sufficient for the investments in the infrastructure to be profitable.

In addition, we observe that the smaller the taxation rate as well as the price index, and at the same time, the higher the labor wages and the investment costs as well as the closer the production elasticity with respect to the labor to 1 (or, the same, the closer the production elasticity with respect to the infrastructure capital to zero), the tighter the range $[\hat{K}, \bar{K}]$ and thereby it is less likely that there exists an initial capital $K_0 < \bar{K}$, such that the investment in the infrastructure is sustainable.

Finally, using the solution found, we can now define optimal taxation. For example, if $K_0 > \bar{K}$, then from Eq. (12a) we have $\bar{K}^{\alpha/(1-\beta)}\bar{\varphi}(\beta^{-1} - 1) = \pi/1 - \tau^*$. Consequently,

$$\tau^* = 1 - \frac{\pi}{\bar{K}^{\alpha/(1-\beta)}\bar{\varphi}(\beta^{-1} - 1)}.$$

(Recall that $L^*(t) = K^{\alpha/(1-\beta)}(t)\varphi(t)$ and in steady state it is a part of the equilibrium found.) A general solution for $\alpha + \beta \neq 1$ and $K_0 > \bar{K}$ is illustrated in Fig. 1 below.ref SHAPE

From a practical viewpoint, it is very easy to find the optimal steady-state investment and then substitute this solution into the constraints to verify whether the investment ensures sustainability. If the constraints are met at the steady state, then the government strategic goals will be eventually met if the initial infrastructure capital is sufficiently large. The steady state, however, does not always exist as shown in the following lemma.

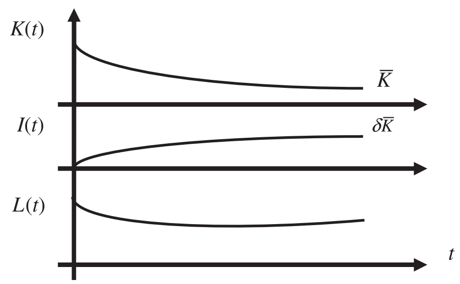


Fig. 1. The optimal equilibrium over time, the case of $K_0 > \bar{K}$.

**Lemma 3.** *Consider the game* (1)–(4). *If* $C_{\mathrm{I}}(F(d(t))) \leqslant \tau b^{\alpha/(1-\beta)}(t)\varphi(t)(\beta^{-1}-1)$, *then no steady state is attainable*, $b(t)$, $d(t)$ *satisfy System B* (*see Appendix*), *and the open-loop Stackelberg equilibrium is* $K^*(t) = b(t)$ *and* $I^*(t) = F(d(t))$ *for* $t \geqslant 1430$.

**Example 2.** Let again $C_{\mathrm{I}}(I) = I^2$ and $\alpha 3 + \beta 3 = 1$, but assume that the labor cost increases slower than the price index so that $\omega(t) = [p(t)/c_{\mathrm{L}}^{\beta}(t)]^{1/(1-\beta)} = \mathrm{e}^{\varepsilon t}$, $\varepsilon 3 < r$. Then from Lemma 3,

$$\dot{d}(t) = -\mathrm{e}^{-(r-\varepsilon)t}\tau\frac{\alpha}{\beta}[a\beta]^{1/(1-\beta)} + \delta\, d(t),$$

$$\lim_{t\to\infty} d(t) = \frac{\partial C_{\mathrm{I}}(I(t))}{\partial I}\mathrm{e}^{-(r-\varepsilon)t} = 0,$$

i.e.,

$$d(t) = \frac{\mathrm{e}^{-(r-\varepsilon)t}}{r-\varepsilon+\delta}\tau\frac{\alpha}{\beta}(a\beta)^{1/(1-\beta)}.$$

Using $d(t)$,

$$\dot{b}(t) = -\delta b(t) + \frac{1}{2}d(t)\mathrm{e}^{rt} = -\delta b(t) + \frac{\tau(\alpha/\beta)(a\beta)^{1/(1-\beta)}\mathrm{e}^{\varepsilon t}}{2(r-\varepsilon+\delta)}.$$

The solution of this differential equation is

$$b(t) = \frac{\tau(\alpha/\beta)(a\beta)^{1/(1-\beta)}\mathrm{e}^{\varepsilon t}}{2(r-\varepsilon+\delta)(\varepsilon+\delta)} + B\mathrm{e}^{-\delta t}.$$

Taking into account that $b(0) = K_0$, we have

$$B = K_0 - \frac{\tau(\alpha/\beta)(a\beta)^{1/(1-\beta)}}{2(r-\varepsilon+\delta)(\varepsilon+\delta)}.$$

Thus,

$$K^*(t) = \frac{\tau(\alpha/\beta)(a\beta)^{1/(1-\beta)}(\mathrm{e}^{\varepsilon t} - \mathrm{e}^{-\delta t})}{2(r-\varepsilon+\delta)(\varepsilon+\delta)} + K_0\mathrm{e}^{-\delta t}$$

and

$$I^*(t) = \frac{\tau(\alpha/\beta)(a\beta)^{1/(1-\beta)}\mathrm{e}^{\varepsilon t}}{4(r-\varepsilon+\delta)} \quad \text{for} \quad t \geqslant 0,$$

which implies increasing (strong) investments in time to compensate domination of the price index increase over labor cost. Fig. 2 presents a solution of this type.

So far we assumed that constraint (11a) is met and discussed the sensitivity of this constraint to system parameters (see Example 1). If the constraint does not hold, then there must be switching points characterizing periods such that the constraint becomes active, i.e., transforms from inequality into the equality. The following lemma shows how constraint (11a) affects the optimal solution if a steady state exists (see Lemma 2). Since the lemma is too technical, we present it without a proof to only explain how to handle a situation when there is an interval of time with the constraint being active.

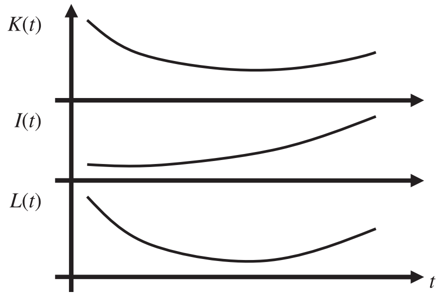


Fig. 2. The optimal equilibrium over time, the case of $\omega(t) = \mathrm{e}^{\varepsilon t}$.

**Lemma 4.** *Consider the differential game* (1)–(4). *Let there exist an interval of time* $t_3 \leqslant t < t_2$, *such that constraint* (11a) *holds as equality*, $\varphi(t) = \bar{\varphi}$ *for* $t \geqslant 1430$, $K_0 \neq \bar{K}$, $b(t)$, $d(t)$, $b_1(t)$, $d_1(t)$, $b_2(t)$, $d_2(t)$ *and* $t_2, t_3$ ($t_3 < t_2$) *satisfy System C* (*see Appendix*).

Then the open-loop Stackelberg equilibrium is $K^*(t) = b(t)$ and $I^*(t) = F(d(t))$ for $0 \leqslant 143t < t_3$, $K^*(t) = b_1(t)$ and $I^*(t) = F(d_1(t))$ for $t_3 \leqslant 143t < t_2$, $K^*(t) = b_2(t)$ and $I^*(t) = F(d_2(t))$ for $t \geqslant 143t_2$, and $\lim_{t\to\infty} K^*(t) = \bar{K}$ and $\lim_{t\to\infty} I^*(t) = \delta\bar{K}$.

Note that graphically the solution described in Lemma 4 looks very similar to that of Lemma 2 (see Fig. 1). The difference between these results is that in the current solution, there is an interval of time $t_3 \leqslant 143t < t_2$ along which the investment and capital trajectories are adjusted (so that constraint (11a) holds as equality) to make the government revenues sustainable. The results obtained so far are altered however when we consider a feedback (closed loop) solution to our differential game. In this case, the investment policy depends on the information available to the public authority.

## 5. The closed-loop equilibrium analysis

So far we have considered only open-loop Stackelberg investment strategies. However, dynamically changing open-loop solutions are difficult to control compared with feedback policies. In what follows, we show how to obtain a closed-loop equilibrium in the conditions of Lemma 2, i.e., when a steady state is attainable. The derivation is accomplished by employing an equivalent formulation of the maximum principle. Specifically, let $\Psi(t) = \psi(t)\mathrm{e}^{rt}$. Then $\psi(t) = \Psi(t)\mathrm{e}^{-rt}$ and

$\dot{\psi}(t) = e^{-rt}(\dot{\Psi}(t) - r\Psi(t))$. Using these notations in conditions of Lemma 2, the co-state equation (14b) and the optimality conditions (19) and (20) take the following form, respectively:

$$\dot{\Psi}(t) - r\Psi(t) = -\tau\frac{\alpha}{\beta}K^{(\alpha/(1-\beta))-1}\varphi(t) + \delta\Psi(t),$$
$$\lim_{t\to\infty}\Psi(t)e^{-rt} = 0, \tag{21}$$

$$\Psi(t) = \frac{\partial C_I(\gamma)}{\partial I}, \tag{22}$$

which defines $\gamma 3(t) = F(\Psi 3(t))$. Consequently, the steady-state conditions are $\dot{K}(t_1) = 0$ and $\dot{\Psi}(t_1) = 0$, which with respect to (21) result in the steady co-state,

$$\bar{\Psi} = \frac{1}{r+\delta}\tau\frac{\alpha}{\beta}\bar{K}^{(\alpha/(1-\beta))-1}\bar{\varphi},$$

as well as steady-state capital $\bar{K}$ and investment $\bar{I} = \delta\bar{K}$ (see Lemma 2). Let us introduce a new function, $\Phi 3(.)$,

$$\Psi(t) = \Phi(K(t)). \tag{23}$$

To simplify the presentation, we next suppress index $t$ wherever the dependence on time is obvious. Then from (23) we have $\dot{\Psi} = \Phi'(K)\dot{K}$, which with respect to (21)–(23) and (1) leads to

$$-\tau\frac{\alpha}{\beta}K^{(\alpha/(1-\beta))-1}\bar{\varphi} + (\delta + r)\Phi(K)$$
$$= \Phi'(K)[F(\Phi(K)) - \delta K], \quad \Phi(\bar{K}) = \bar{\Psi}.$$

Thus, we have proved the following theorem.

**Theorem 1.** *Consider the differential game* (1)–(4), *with a Cobb–Douglas production function* $f(K, L) = aK^{\alpha 3}L^{\beta 3}$. *Assume conditions of Lemma* 2. *Then the time-invariant feedback Stackelberg equilibrium is*

$$I^* = F(\Phi(K)), \tag{24}$$

*where* $\Phi 3(K)$ *satisfies the following differential equation*:

$$-\tau\frac{\alpha}{\beta}K^{(\alpha/(1-\beta))-1}\bar{\varphi} + (\delta + r)\Phi(K)$$
$$= \Phi'(K)[F(\Phi(K)) - \delta K], \quad \Phi(\bar{K}) = \bar{\Psi}. \tag{25}$$

**Example 3.** Consider the case of $C_I(I) = c_I I^2$. Then $\bar{K} = \left[(2c_I\delta(r+\delta)\beta)/\tau\alpha\bar{\varphi}\right]^{(1-\beta)/(\alpha+2\beta-2)}$ and from (22) we have $I^* = \Psi/2c_I = \Phi(K)/2c_I$, which with respect to (25) implies that $\Phi 3(K)$ satisfies the backward differential equation

$$\Phi'(K)\left[\frac{\Phi(K)}{2c_I} - \delta K\right] - (\delta + r)\Phi(K) + \xi K^{(\alpha/(1-\beta))-1} = 0,$$
$$\Phi(\bar{K}) = \frac{\xi}{r+\delta}K^{(\alpha/(1-\beta))-1},$$
$$\bar{K} = \left[\frac{\xi}{2c_I\delta(r+\delta)}\right]^{(1-\beta)/(-\alpha-2\beta+2)},$$

where $\xi = \tau\alpha\bar{\varphi}/\beta$. We solve this equation with Maple for $\alpha 3 = 0.1$, $\beta 3 = 0.1$, $w = 10$, $\tau 3 = 0.4$, $\delta 3 = 0.04$, $r = 0.002$ and $c_I = 1$.

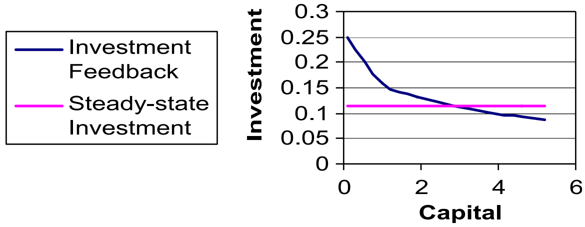


Fig. 3. Feedback equilibrium investment as a function of capital, $I^* = F(\Phi(K))$.

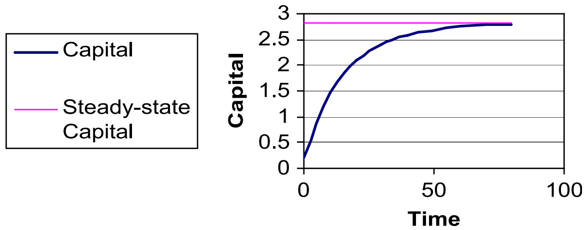


Fig. 4. Evolution of capital over time, $K^*(t)$.

The resultant feedback policy, $I^* = \Phi(K)/2c_I$, is illustrated graphically in Fig. 3. The corresponding evolution in time of the capital and investment for the case of $K(0) = 0.2 < \bar{K} = 2.830428$ are depicted in Figs. 4 and 5, respectively.

From Figs. 3–5 we observe that the greater the capital, the lower the investments. Note that when the infrastructure capital is greater (smaller) than the steady-state level $\bar{K}$, it is optimal to invest less (more) than $\bar{I} = \delta\bar{K} = 0.113217$, so that the overall accumulated capital decreases (increases) towards the steady state. Furthermore, the investments decrease much faster when the capital exceeds the steady-state level compared with the rate of their decrease when the capital is lower than the steady-state level (see Fig. 3).

Finally, note that according to Eq. (6), the optimal employment solution is apparently a feedback policy, is $L^*(t) = \varphi K^{\alpha/(1-\beta)}(t)$. Since the unit

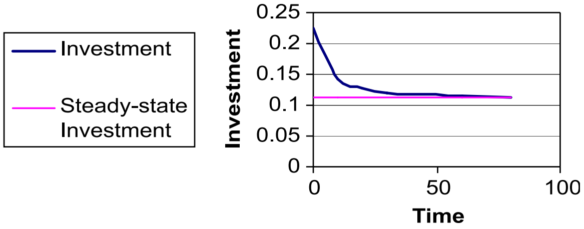


Fig. 5. Evolution of investment over time, $I^*(t)$.

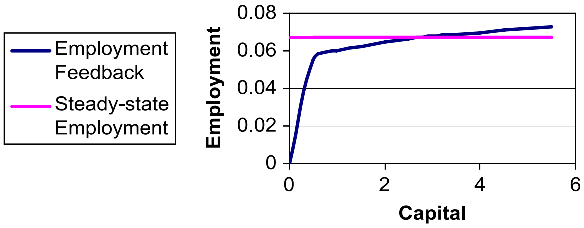


Fig. 6. Feedback equilibrium employment as a function of capital.

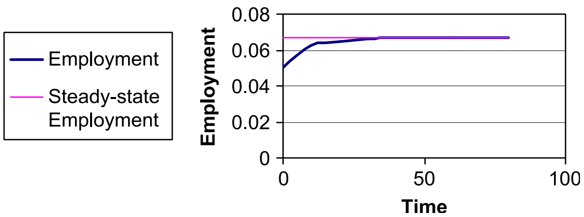


Fig. 7. Evolution of employment over time, $L^*(t)$.

investment cost $c_L$ is constant in our example, the steady-state capital induces the equilibrium employment to attain a steady state, $\bar{L} = \bar{\varphi}\bar{K}^{\alpha/(1-\beta)}$. The optimal employment feedback policy and the evolution in time of employment for our example are shown in Figs. 6 and 7, respectively. From these figures we observe that the employment increases with the capital (Fig. 6) and it tends to the steady-state employment level $\bar{L} = 0.067227$ (Fig. 7). The rate of employment changes much faster when the infrastructure capital is low, it increases towards the steady-state labor when $K(0) < \bar{K}$ and decreases when $K(0) > \bar{K}$.

## 6. Conclusion

In a "capital primitive" economy, infrastructure investments are supplied by the government and in some cases by international economic agencies or foreign governments. In such cases, employment growth rate can be ensured if both taxation rates and capital investments are sustainable. This paper has addressed these issues in "a Stackelberg game–optimal control framework" by considering a "capital-primitive" labor economy. The results we have obtained are tractable, providing some preliminary insights regarding the strategic policies that governments in such economies can sustain. One possibility to ensure the strategic goals is to retain steady investments. We show that there exists an optimal steady-state investment policy. Furthermore, for a wide range of problem parameters the lower the production elasticity with respect to the infrastructure, the lower the steady-state investment and infrastructure capital. On the other hand, if the production elasticity with respect to the labor is close to 1, then the lower the elasticity with respect to the infrastructure, the higher the steady-state investment and infrastructure capital.

We provide both open-loop and feedback equilibrium strategies to attain an optimal steady-state investment policy. Analysis of the feedback equilibrium shows that the greater the capital, the lower the investments. When the infrastructure capital is greater (smaller) than the steady-state level $\bar{K}$, it is optimal to invest less (more) than $\delta\bar{K}$, so that the overall accumulated capital decreases (increases) towards the steady state. Furthermore, the investment rates decrease much faster when the capital exceeds the steady-state level compared with the rate of their decrease when the capital is lower than the steady-state level.

To maintain the "control tractability" of our problem, a number of important factors and issues were neglected. For example, investment can be financed not only by tax collection but by borrowing as well. While in some cases, borrowing may be attractive in the short run it can also be non-sustainable. For this reason, issues associated with how much and at what price to borrow are clearly worth considering. We have also neglected the potential to finance investment in infrastructure through the market mechanism as well as through self-investment by firms. This is, of course, an important issue to reckon with. However, such opportunities occur only in economies that have capital markets on the one hand (where firms and the government can issue bonds for infrastructure investment) and have firms capable of investing in capital on the other. While this is the case in most developed and semi-developed countries, it is not the case in the type of economies we refer to. Of course, an extension of this paper will consider these issues as well and deal with the important questions many governments face today—financing infrastructure projects or seeking agreements with firms (through BOT contracts for example) to invest in infrastructure capital. Tax breaks of various sorts applied to investments in pollution abatement are a case in point where firms are given an incentive to proceed to such investments. While we have not dealt explicitly with these problems, the "control" framework set in this paper provides an opportunity to deal theoretically with many of such issues.

## Appendix

**Proof of Lemma 1.** Consider the following solution for the state and co-state variables, which is characterized by a sustained constant level of capital $K(t)=\bar{K}$:

$$\dot{\psi}(t)=-\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}K^{(\alpha/(1-\beta))-1}(t)\varphi(t)+\delta\psi(t), \quad (A.1)$$

$$\frac{\mathrm{d}K(t)}{\mathrm{d}t}=-\delta K(t)+I(t)=0. \quad (A.2)$$

This solution is feasible if constraint (11a) holds and is optimal if the optimality conditions (19), (20) hold, i.e., if $\dot{\psi}(t)=\left(\left(\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I\right)r\mathrm{e}^{-rt}\right)'_t=-\left(\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I\right)r\mathrm{e}^{-rt}$, which with respect to (A.1) results in

$$-\frac{\partial C_{\mathrm{I}}(\delta\bar{K})}{\partial I}r\mathrm{e}^{-rt}=-\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}\bar{K}^{(\alpha/(1-\beta))-1}(t)\varphi(t)+\delta\psi(t),$$

i.e.,

$$-\frac{\partial C_{\mathrm{I}}(\delta\bar{K})}{\partial I}r\mathrm{e}^{-rt}=-\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}\bar{K}^{(\alpha/(1-\beta))-1}(t)\varphi(t)+\delta\mathrm{e}^{-rt}\frac{\partial C_{\mathrm{I}}(\delta\bar{K})}{\partial I}.$$

Substituting $\bar{\varphi}=\varphi(t)$ into both last expression and constraint (11a), we obtain the equation and the condition stated in the lemma. Finally, we straightforwardly verify that the boundary condition from (14c), $\lim_{t\to\infty}\psi(t)=\lim_{t\to\infty}\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I\,\mathrm{e}^{-rt}=0$ holds. □

**Proof of Lemma 2.** Consider now a solution that tends to the steady investment policy, $K(t)=\bar{K}$ and $I(t)=\delta\bar{K}$ for $t\to\infty$. Then substituting $\gamma=F(\psi(t))$ from (20) into (1), we have

$$\dot{K}(t)=-\delta K(t)+F(\psi(t)),\quad K(0)=K_0,\quad \lim_{t\to\infty}K(t)=\bar{K}. \quad (A.3)$$

Differentiating (A.3) we obtain

$$\ddot{K}(t)=-\delta\dot{K}(t)+\frac{\partial F(\psi(t))}{\partial\psi(t)}\dot{\psi}(t), \quad (A.4)$$

which along with (A.1) constitutes a system of two differential equations in two unknowns, $K(t)$ and $\psi(t)$, with $\lim_{t\to\infty}\psi(t)=0$, as stated in the lemma. The solution to this system satisfies the optimality conditions and is feasible if constraint (11a) is not violated,
$C_{\mathrm{I}}(F(\gamma))\leqslant\tau b^{\alpha/(1-\beta)}(t)\varphi(t)[\beta^{-1}-1]\quad \varphi(t)=\bar{\varphi}$. □

**Proof of Lemma 3.** The fact exists that if $\varphi(t)$ is time dependent, then no steady-state solution exists is immediate. Indeed, equation $\left(\partial C_{\mathrm{I}}(\delta\bar{K})/\partial I\right)(r+\delta)=\tau(\alpha/\beta)\bar{K}^{(\alpha/(1-\beta))-1}\bar{\varphi}$ derived in Lemma 1 under the assumption that $\varphi(t)=\bar{\varphi}$ and $K(t)$ are constant, $K(t)=\bar{K}$, no longer hold if $\varphi(t)$ is a function of time (namely that the ratio of prices to labor costs are not maintain at a fixed proportion). The system of equations is similar to that of Lemma 2.

*System A (for Lemma 2)*:

$$\ddot{b}(t)=-\delta\dot{b}(t)+\frac{\partial F(d(t))}{\partial d(t)}\dot{d}(t),$$
$$b(0)=K_0,\quad \lim_{t\to\infty}b(t)=\bar{K};$$

$$\dot{d}(t)=-\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}b^{(\alpha/(1-\beta))-1}(t)\varphi(t)+\delta d(t),\quad \lim_{t\to\infty}d(t)=0.$$

*System B (for Lemma 3)*:

$$\dot{b}(t)=-\delta b(t)+F(d(t)),\quad b(0)=K_0;$$

$$\dot{d}(t)=-\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}b^{(\alpha/(1-\beta))-1}(t)\varphi(t)+\delta d(t),\quad \lim_{t\to\infty}d(t)=0.$$

*System C (for Lemma 4)*:

$$\ddot{b}(t)=-\delta\dot{b}(t)+\frac{\partial F(d(t))}{\partial d(t)}\dot{d}(t),\quad b(0)=K_0,$$

$$b(t_3)=\left[\frac{C_{\mathrm{I}}(F(d(t_3)))}{\tau\bar{\varphi}(\beta^{-1}-1)}\right]^{(1-\beta)/\alpha},$$

$$\dot{d}(t)=-\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}b^{(\alpha/(1-\beta))-1}(t)\varphi(t)+\delta d(t),\quad d(t_3)=\frac{\partial C_{\mathrm{I}}(F(d(t_3)))}{\partial I}\mathrm{e}^{-rt_3};$$

$$b_1(t) = \left[\frac{C_{\mathrm{I}}(F(d(t))}{\tau\bar{\varphi}(\beta^{-1}-1)}\right]^{(1-\beta)/\alpha}, \ \lambda(t) = \frac{d_1(t)}{\partial C_{\mathrm{I}}(t)/\partial I(t)} - \mathrm{e}^{-rt},$$

$$\dot{d}_1(t) = -\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}b_1^{(\alpha/(1-\beta))-1}(t)\bar{\varphi} - \lambda(t)\tau\frac{\alpha}{\beta}b_1^{(\alpha/(1-\beta))-1}(t)\bar{\varphi} + \delta d_1(t), \quad d(t_2) = \frac{\partial C_{\mathrm{I}}(F(d_1(t_2))}{\partial I}\mathrm{e}^{-rt_2},$$

$$\ddot{b}_2(t) = -\delta\dot{b}(t) + \frac{\partial F(d_2(t))}{\partial d_2(t)}\dot{d}_2(t),$$

$$\lim_{t\to\infty}\dot{b}_2(t) = \bar{K} + \delta\bar{K}, \ \lim_{t\to\infty}\dot{b}_2(t) = \bar{K},$$

$$\dot{d}_2(t) = -\mathrm{e}^{-rt}\tau\frac{\alpha}{\beta}b_2^{(\alpha/(1-\beta))-1}(t)\bar{\varphi} + \delta d_2(t),$$

$$\lim_{t\to\infty} d_2(t) = 0. \qquad \square$$

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