A network approach to modeling the multi-echelon spare-part inventory system with backorders and interval-valued demand

Eugene Levner a,b,∗, Yael Perlm ana, T.C.E. Cheng c, Ilya Levnera

a Department of Management, Bar-Ilan University, 52900 Ramat-Gan, Israel
b School of Economics, Ashkelon Academic College, Ashkelon, Israel
c Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Kowloon, Hong Kong

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A B S T R A C T

A multi-echelon inventory system implies the existence of a hierarchy of stocking locations, and the dependence and interaction between them. We consider a multi-echelon, spare-part inventory management problem with outsourcing and backordering. The problem is characterized by deterministic repair time/cost, and supply and demand that lie within prescribed intervals and that vary over time. The objective is to minimize the total inventory and transportation costs. We develop a network model for problem analysis and present a network flow algorithm for solving the problem. We prove that the Wagner–Whitin property, known for the lot-sizing problem, can be extended to the spare-part inventory management problem under study.

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1. Introduction

For many expensive technological systems, such as computer systems, medical equipment, and military defence systems, high availability of spare parts is essential. As the spare parts can be expensive, it is often more economical to repair them than to replace them. Timely supply of spare parts can be achieved by holding sufficient stocks and/or minimizing the repair work. A trade-off between spare-part inventories and repair facilities is achieved by increasing spare-part inventories and decreasing the repair capacity, and vice versa. Thus an optimization problem concerning the optimal investment allocation to both spare-part inventories and repair capacity arises.

Inventory systems where units that fail are repaired at a repair shop rather than discarded are called repairable-item inventory systems. A repairable spare-part network implies the existence of locations where spare parts are stocked and that there are facilities to repair failed items. In this paper we consider a multi-echelon inventory system with several operational sites (the bases) and two supply modes, namely an external supplier and a repair shop (called the depot). A failed item is sent from its base to the repair shop and the repaired item is returned to the originating base. We investigate the repairable-item inventory management problem from the perspective of network analysis, while past studies have taken disparate approaches to treat the problem (see, e.g., the surveys by Axsäter (1990), Díaz and Fu (1995), Kennedy et al. (2002), Minner (2003), and Sleptchenko et al. (2005), and the numerous references contained in them).

There is extensive literature that addresses spare-part inventory systems with random demand under different assumptions on the data probability distributions. Hausman and Scudder (1982), Pyke (1990), Verrijdt et al. (1998), Perlman et al. (2001), Sleptchenko et al. (2002), Sleptchenko et al. (2005), Perlman and Kaspi (2007), and Adan et al. (2009) study stochastic multi-echelon inventory systems with several repair modes. All of these models assume that failures occur according to a Poisson process with a constant rate. However, in many practical situations there is no convincing argument to justify the latter assumption, especially if the repair process is not steady. On the other hand, for almost any practical situation, the inventory manager can give an accurate deterministic estimate of the lower and upper bounds on the inventory level in any period over the planning horizon. We study such type of deterministic bounded (i.e., interval-valued) demand in this paper.

To the best of our knowledge, not much has been done to date on modeling the planning and scheduling of spare-part inventories with deterministic demand. Most of the related studies focus on the very special case involving a single supplier and a fixed-demand inventory (e.g., Prager, 1956; Federgruen, 1993; Gass, 2003; Abdul-Jalbar et al., 2006; Federgruen et al., 2007; Perlman and Levner, 2010). The models are generally formulated as a cost minimization problem, with a cost function comprising the holding cost, ordering/setup cost, and either an explicit penalty cost or a specified service level constraint. The models in the literature can be divided into supply with or without outsourcing, and backorders versus lost sales, while the models with backorders can be further divided into cost versus
service level requirements. Such models have been applied to handle production planning and scheduling in practice. They often employ lot-sizing algorithms, ranging from simple heuristics to complicated dynamic programming, network flow formulation, and integer programming, as solution methods. A multi-echelon inventory model implies the existence of a hierarchy of stocking locations and interaction among their inventories.

In this paper we study a repair shop operating under the assumption that supply and demand in the forthcoming periods are predictable and deterministic. This allows us to determine the optimal safety stock at the depot and the optimal levels of orders of spare parts from the external supplier. We suggest taking a network approach to model the multi-echelon, multi-supplier problem. The solution method extends the early deterministic network approach to model the multi-echelon, multi-supplier of spare parts from the external supplier. We suggest taking a network approach to model the multi-echelon, multi-supplier problem. The solution method extends the early deterministic network approach to model the multi-echelon, multi-supplier problem. The solution method extends the early deterministic network approach to model the multi-echelon, multi-supplier problem.

We consider a repairable-item inventory system in which the number of daily failures at each base may change from day to day. We call these entering (failed) items supplies. Similarly, we assume that the demand for good items, which are required at each base every day, also change from day to day. Both the daily supply $R_t(k)$ and daily demand $F_t(k)$ can be predicted, but up to a limited degree of precision that is known in advance. Specifically, we assume $R_t(k) = R^0_t(k) + \Delta R^0_t(k)$ and $F_t(k) = F^0_t(k) + \Delta F^0_t(k)$, where $R^0_t(k)$, $F^0_t(k)$, $\Delta R^0_t(k)$, and $\Delta F^0_t(k)$ are known entities. It follows that, for each base $k$, there are given prescribed intervals, called the supply window: $R^0_t(k) = [\underline{R}^0_t(k), \overline{R}^0_t(k)]$ and the demand window: $F_t(k) = [\underline{F}_t(k), \overline{F}_t(k)]$, in which the daily supply and daily demand lie, respectively. Here the lower bound $\underline{F}_t(k)$ is a forecast of the minimum demand for good items that should be available at base $k$ on day $t$. On the other hand, the upper bound $\overline{F}_t(k)$ is the maximum demand allowed at base $k$ on day $t$. If the stock required to fulfill this demand is insufficient on a certain day, the unfulfilled demand is backordered. The backorder is fulfilled later when new items arrive from the external supplier or when the repair shop fixes the failed items. The expected number of backorders, as well as the backorder cost and the number of backorder days, is an important measure of the effectiveness of inventory management as discussed by Liberopoulos et al. (2010). If the number of entering items plus the available stock at the depot are insufficient to fulfill the total demand, then the items are outsourced from the external supplier, which incurs a higher cost.

The objective of our study is to develop a model to determine the optimal number of spare parts to purchase from the external supplier and the optimal number of failed items to be repaired daily (and stored if necessary) in order to meet the required demand at the minimum operating cost.

Our model has three main features. First, we assume supply and demand are predictable up to a limited degree of precision. For this situation we formulate a network-flow model with lower and upper flow bounds to optimize the circulation of spare parts and decide the repair option. Second, we explicitly incorporate transportation time/cost and repair time/cost into the repair process. Third, we implicitly introduce multiple suppliers, as well as backorder time and penalty into the network model.

The rest of the paper is organized as follows: In the next section we present a mathematical model of the problem under study and show that it is equivalent to a minimum-cost network flow problem on a special network with lower and upper bounds. In Section 3 we study the structural properties of the optimal solution and decompose the initial problem into two independent sub-problems. In Sections 4 and 5 we present a solution algorithm and provide several numerical examples to illustrate the solution procedure, respectively. We conclude the paper and suggest topics for future research in Section 6.

2. The model

In this section we describe the spare-part supply system under study, introduce the parameters and variables, and formulate the objective function and constraints.

2.1. System description

We consider the scenario in which there are several operational sites (bases) served by a repair shop (depot) with a storage facility, where spare parts are kept. The stock at the depot can also be filled from an external supplier (see Fig. 1 borrowed from Perlman and Levner, 2010).

When there is demand for the spare part at a base, if there is sufficient stock in the depot, replacement items are sent through

![Fig. 1. Flow of parts in a multi-echelon system (Perlman and Levner, 2010).](image-url)
the out-pipeline to meet the demand at the base. Otherwise the demand is backordered. On the other hand, the failed items are sent to the depot for repair through the in-pipeline. We assume that the depot provides a multi-modal repair service, i.e., the repair time ranges from $t_{\text{min}}$ days when items are repaired at the fastest rate to $t_{\text{max}}$ days when items are repaired at the slowest rate. The fast repair service is more expensive because of increased manpower cost from either hiring additional personnel or paying existing personnel to work additional shifts. We further assume that a repaired item is as good as new, which can be used to meet the demand at a base on a certain day, fill a backorder if one exists, or become part of the stock at the depot. In addition, there is no distinction between a repaired item arising from the multi-modal repair service and a new item purchased from the external supplier, i.e., they all become part of the depot stock. We also assume that there is infinite repair capacity at the depot and that the depot can repair every failed item. Items may also be purchased from the external supplier at a price much higher than the unit repair cost and sent to the depot through the purchase pipeline.

### 2.2. Notation and variables

Let $t$ denote a period in the planning horizon ($t=1,...,T$), $k$ a base ($k=1,...,K$), and $i$ the repair mode ($i=1$ for fast repair and $i=2$ for slow repair).

We assume that the following parameters are given as input data:

- $R^l_t(k)$, $R^u_t(k)$—lower and upper bounds on the number of good items required on day $t$ at base $k$, respectively.
- $F^l_t(k)$, $F^u_t(k)$—lower and upper bounds on the entering flow (supply) of failed items on day $t$ at base $k$, respectively.
- $c_1$—unit cost charged for transporting an item.
- $c_2$—unit cost charged for distributing an item.
- $c_3$—unit cost charged for repairing an item by the fast service.
- $c_4$—unit cost charged for repairing an item by the slow service.
- $c_5$—unit price for buying a new item.
- $c_6$—unit inventory holding cost.
- $c_7$—unit penalty cost for backordering.
- $T$—planning horizon.
- $T_1$—shipping time at the in-pipeline.
- $T_2$—shipping time at the out-pipeline.
- $T_3$—lead time from the external supplier to the depot.
- $T_4$—fast repair time at the depot.
- $T_5$—slow repair time at the depot.

We introduce the following (integer) variables:

- $R_t(k)$—daily demand, i.e., the number of good items required on day $t$ at base $k$.
- $F_t(k)$—daily supply, i.e., the number of failed items available on day $t$ at base $k$.
- $q_t(k)$—number of failed items sent from base $k$ on day $t$.
- $l_t(k)$—number of failed items left at base $k$ on day $t$.
- $Q_t(k)$—flow of failed items that arrive at the depot from base $k$ on day $t$.
- $DS_t$—depot stock of good items on day $t$.
- $x_t$ (0 $\leq y_t$) —number of items sent for fast (slow) repair service on day $t$.
- $X_t$—number of items repaired by the fast repair service on day $t$.
- $Y_t$—number of items repaired by the slow repair service on day $t$.
- $U_t$—number of items purchased from the external supplier on day $t$.
- $u_t$—number of items ordered from the external supplier on day $t$.

### 2.3. Constraints and objective function

The following are the constraints on the flow of items in the network:

**Constraint 1.** Lower and upper bounds on the number of failed items at a base:

$$R^l_t(k) \leq F_t(k) \leq R^u_t(k).$$

**Constraint 2.** Balance of failed items at a base:

$$F_t(k) + l_{t-1}(k) = q_t(k) + l_t(k).$$

**Constraint 3.** Transportation of failed items through the in-pipeline:

$$Q_t(k) = (L_t(k) + B_t(k)).$$

**Constraint 4.** Integrating and distributing failed items from different bases:

$$\sum_t Q_t(k) = x_t(k) + y_t(k); \quad X_t(k) = x_{t-1} - u_t; \quad Y_t(k) = y_{t-1} - u_t.$$

**Constraint 5.** Purchasing a new item from the external supplier:

$$U_t = u_{t-1}.$$

**Constraint 6.** Balance of the depot stock:

$$X_t + Y_t + U_t + DS_{t-1} = DS_t + S_t.$$

**Constraint 7.** Distribution of good items from the depot through different out-pipelines to different bases:

$$S_t = \sum_z z_t(z).$$

**Constraint 8.** Transportation of good items through the out-pipeline:

$$Z_{t+m}(t) = Z_t(k).$$

**Constraint 9.** Balance of good items at the base:

$$Z_t(k) = B_t(k) = R_t(k) + B_{t-1}(k).$$

**Constraint 10.** Lower and upper bounds on the number of good items at a base:

$$R^l_t(k) \leq R_t(k) \leq R^u_t(k).$$

Our objective is to minimize the total cost of transporting, distributing, repairing, holding, purchasing, and backordering of items over the planning horizon. So we wish to Minimize:

$$c_1 \sum_t \sum_k (q_t(k) + Z_t(k)) + c_2 \sum_t \sum_k (Q_t(k) + Z_t(k))$$

$$+ c_3 \sum_t x_t + c_4 \sum_t y_t + c_6 \sum_t DS_t + c_5 \sum_t u_t + c_1 \sum_t BO_t(k).$$

Notice that interval bounds similar to (1) and (10) can be imposed on other variables, e.g., shipping items through the in-pipeline and the out-pipeline.
2.4. Network flow model

The linear programming problem formulated in Section 2.3 is equivalent to a minimum-cost network flow problem on a specially constructed network with lower and upper capacity bounds on its arcs (see Ford and Fulkerson (1962) for discussions on networks with upper and lower bounds). We construct an underlying network as follows:

1. For each day \( t = 1, \ldots, T \), define the base supplies as \( K \) “source links”. There are \( KT \) sources in total.
2. For each day \( t = 1, \ldots, T \), define the base demands as \( K \) “sink links”. There are \( KT \) sinks in total. Two numbers assigned to the source and sink links denote the lower and upper bounds on the supply and on the demand at the corresponding base.
3. For each day \( t = 1, \ldots, T \), define separate nodes corresponding to the state of the in-pipeline, out-pipeline, depot, and external supplier.
4. For every pair of nodes, define an arc if (and only if) there is a flow of items from one node to the other and impose the relevant linear constraints in Section 2.3 corresponding to the flow balance condition on each intermediate node.
5. Associate costs (of transportation, repair, and inventory holding) to the corresponding arcs.
6. The flows of items moving bottom-up in the most-right network columns (corresponding to the base demands) do not physically move “against time” but rather are stored at the bases and correspond to the backlogged items.

Fig. 2 presents the network formulation of the problem for the special case where there are two bases. The time parameters are given in Table 1.

3. Model analysis

If the outsourcing cost is high and exceeds the repair cost on any day \( t \), then it is more economical to repair than to purchase new parts from the external supplier. It follows that in this case, after day \( T_1 + T_5 \) there is no need to consider the link between the outsourcing supplier and the depot. Then, without loss of generality, the nodes in the network corresponding to the external supplier after day \( T_1 + T_5 \) can be deleted. For instance, in Example 1 below, in Fig. 4 it is possible to delete the link leading from node 20 to node 16 and after that, delete node 20.

At the same time, if the depot stock on day 1 is limited and not replenished, then its level will be at most \( DS_1 \) on day \( T_1 + T_5 \) and it will be exhausted on one of the next few days, \( T_0 \), which is defined as follows:

\[
DS_1 \geq \sum_{k = 1}^{K} \sum_{t = T_1 + T_5}^{T_1 + T_5 + T_4} r_t(k) \quad \text{and} \quad DS_1 < \sum_{k = 1}^{K} \sum_{t = T_1 + T_5 + T_4 + 1}^{T_1 + T_5 + T_4 + T_5} r_t(k)
\]

It follows that there is no need to establish a link between the depot stock on day \( T_0 \) and the depot stock on day \( T_0 + 1 \). In the considered example, this is a link between node 15 and node 16, whereas \( T_0 = 3 \).

Consider now the situation where backlogging is prohibited. Then the network corresponding to the echelons from the depot stock to (demanding) bases \( 1, \ldots, K \) at time from day 1 to day \( T_0 + 1 \) will be isolated from the entire network. The same is true when backlogging is permitted but its cost is sufficiently high.
Fig. 3. Network flow of Example 1.

Fig. 4. Decomposition of the network.
Since in this case it is prohibitively costly to use backlogging, so the backward links between any day $t+1$ and day $t$ in the column “Bases Demand” can be deleted.

As a result, the original network can be decomposed into two disjoint sub-networks (see Fig. 4). We draw the following conclusion.

**Claim 1.** If the outsourcing and backlogging costs are high while the depot stock capacity is finite, the optimal solution for the problem defined on the original network can be obtained by decomposing the initial network into two independent sub-networks.

The corollary of this claim is that the optimization sub-problems can be solved independently of each of the obtained sub-networks, whereas an optimal solution for the initial problem is obtained as the union of the latter partial solutions.

For instance, the initial network in Fig. 2 can be decomposed into two sub-networks presented in Fig. 4, in which we add a cutting line to separate the sub-networks. Henceforth, the sub-network on the left of the cutting line is called sub-network #1 and the network on the right of the cutting line is called sub-network #2.

It is obvious that if after decomposition of the original network, some node in the sub-network corresponding to sub-network #2 turns out to be isolated (i.e., there are no outgoing arcs), then this node can be excluded from the network, which further reduces the computational load. For instance, in the following example, node 20 in Fig. 4 is excluded from the sub-network depicted in Fig. 6.

The next claim permits us to further simplify the structure of the network.

**Claim 2.** If, on a certain day $t_0$, it holds that

$$t_0 + T_1 + T_2 + T_3 + T_5 > T,$$

then all the nodes lying below day $t_0$ in the network can be deleted.

Indeed, if a spare part appears in the system on day $t > t_0$, then it can arrive at a base from the out-pipeline only beyond the planning horizon $T$. For instance, in the following example, all the nodes lying below day 6 are excluded from the underlying network.

The following claim extends the well-known Wagner–Whitin property (Wagner and Whitin, 1958).

**Theorem 1.** There exists an optimal solution to the uncapacitated spare part management problem in which at the exit of the depot, $D_{S_t-1}, X_t, Y_t, U_t = 0$ for all $t$, where one and only one term (either $D_{S_t-1}$, $X_t$, $Y_t$, or $U_t$) is non-zero.

**Proof.** Let us introduce a single super-source and a single super-sink to the network presentation of the problem. Add arcs leading from the super-source to all the source nodes in the network and add arcs from all the sink nodes to the super-source. For any feasible solution, the problem reduces to a minimum-cost flow problem (see the network shown in Fig. 2). In any extreme solution, the arcs with positive flow from a super-source to a super-sink form an acyclic subgraph. As far as we are looking for the minimum-cost solution and the network is uncapacitated, the optimal flow from the super-source to any node runs along a single path from the super-source to that node, namely, the path that has the minimum cost among all the paths from the super-source to that node. Then for any node representing the depot in the network (see, e.g., node 17 in Fig. 4) it holds that $D_{S_t-1}, X_t, Y_t, U_t = 0$ for all $t$, where one and only one term (either $D_{S_t-1}$, $X_t$, $Y_t$, or $U_t$) is non-zero.

**Remark.** In the uncapacitated lot-sizing problem considered by Wagner and Whitin (1958), there is neither outsourcing nor repair flow. There are only production lot size $P_t$ in period $t$ and inventory $D_t$ at the end of period $t$. The Wagner–Whitin property claims that in this case there exists an optimal solution to the problem in which $D_{S_t-1}P_t = 0$ for all $t$.

4. **The algorithm**

We propose a network flow algorithm for solving the considered spare part management problem. It begins with reducing the initial minimum-cost flow problem with lower and upper bounds to the minimum cost circulation problem. The problem is solved in three stages. In the first stage, an auxiliary network taking into account only lower bounds is constructed, and an auxiliary flow is found from an additional source to an additional sink in this network. In the second stage, the auxiliary flow is used for finding a feasible flow for the initial minimum-cost flow problem. Finally, in the third stage, the optimal flow is found using an iterative minimum-cost flow procedure.

The algorithm works as follows:

(Initialization). The minimum-cost flow problem with lower and upper bounds reduces to the minimum-cost circulation problem (MCCP). For this purpose, we first introduce the “super-source” $S$ and the “super-sink” $T$, and then add a new arc leading from $T$ to $S$ with a lower bound 0 and an upper bound infinity. Denote this network as $N$.

(Stage 1) Using the obtained MCCP, we construct an auxiliary minimum-cost flow problem $AMCFP$ with given lower bounds for the flows. This stage consists of the following three steps:

(S1.1) Construct a supporting network $N_1$ having the same nodes and arcs as the original network $N$, and the upper flow capacity on arcs equal to the lower bounds in MCCP.

(S1.2) Group all the nodes of $N_1$ into three classes: (a) those where there is a balance of capacities, i.e., the sum of capacities of the entering arcs is equal to the sum of capacities of the outgoing arcs, (b) those nodes where the sum of capacities of the entering arcs is larger than the sum of capacities of the outgoing arcs are called “generating nodes”, and (c) those nodes where the sum of capacities of the entering arcs is less than the sum of capacities of the outgoing arcs are called “absorbing nodes”.

(S1.3) Add an additional source $s'$ and sink $t'$ and, after that, add the arcs leading from $s'$ to all the generating nodes of $N_1$, as well as the arcs leading from the absorbing nodes to the source $t'$. Add the capacities on the latter arcs, which are equal to the corresponding capacity misbalances. After that, we construct a flow from $s'$ to $t'$, the value of which is equal to the total misbalance so that there will be flow balance in all the nodes of the network except $s'$ and $t'$. For all the other arcs, define their upper flow bound as the differences between the upper and lower bounds of the arcs in the original network $N$. Denote this network as $N_2$.

(Stage 2) Using the obtained network $N_2$, find a feasible solution to MCCP. This stage consists of the following two steps:

(S2.1) Find the maximum flow in the obtained network $N_2$ using the standard Ford–Fulkerson maximum-flow algorithm (Ford and Fulkerson, 1962) on a network with zero lower bounds.

(S2.2) Add the lower bounds given on the arcs of $N$ to the obtained minimum-cost flow in the network $N_2(s', t')$. As a result, we obtain a feasible solution to the initial minimum-cost flow problem.

(Stage 3) Starting from the found feasible flow, find an optimal solution to the considered minimum-cost flow problem on a network with the help of the iterative primal minimum-cost algorithm by Klein (1967).
(S3.1). Define the modified costs as follows:

$$c_{ij}' = \begin{cases} 
  c_{ij}, & \text{if } 0 \leq x_{ij} < b_{ij} \\
  \infty, & \text{if } x_{ij} = b_{ij} \\
  -c_{ij}, & \text{if } x_{ij} > 0
\end{cases}$$

(S3.2). Using the found $c_{ij}'$, find a cycle of negative total cost in the network $N_2(s', t')$. If there is no such cycle, then the found flow is optimal (minimum cost). If such a cycle exists, then add the flow along it a value of $d = \min(b_{ij} - x_{ij}, x_{ji})$, where $\min$ is computed for all the arcs of the negative-cost cycle. Then go to Step S3.1.

The iterations of the Klein algorithm are repeated until there is a negative-cost cycle in the network. When there is no negative cycle, it follows that the optimal solution is found. The detection of the negative-cost cycle is done with the help of a standard negative-cycle detector algorithm, e.g., the Bellman–Ford algorithm.

5. Numerical examples

Example 1. For convenience, we first consider a simple example with numerical (non-interval) input data extracted from Perlman and Levner, 2010). There are two bases and the length of the planning horizon is 6 days. Since the requirement for good items is equal to the number of failed items, it follows that $R_t(k) = F_t(k)$, where $R(1) = (5, 6, 7, 8, 7, 9)$ and $R(2) = (6, 7, 10, 8, 5)$.

There is an initial stock of 15 spare items at the depot. Table 1 presents the time parameters, while Table 2 gives the cost parameters. Table 3 gives the optimal solution, which yields the minimum cost $2255.1$. The corresponding optimal distribution of flows in the network is given in Fig. 3.

Example 2. We extend Example 1 to a 6-day planning horizon, assuming that the daily supply and demand are not known precisely but allowed to vary in the following intervals (“windows”): $R_1(1) = F_1(1) = ([4, 6]; [5, 7]; [6, 8]; [7, 9]; [6, 8]; [8, 10])$ and $R_1(2) = F_1(2) = ([5, 7]; [5, 7]; [6, 8]; [9, 11]; [7, 9]; [4, 6])$.

<table>
<thead>
<tr>
<th>Value</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_1 = $0.05</td>
<td>Transportation cost</td>
</tr>
<tr>
<td>c_2 = $0</td>
<td>Distribution cost</td>
</tr>
<tr>
<td>c_3 = $15</td>
<td>Fast repair cost</td>
</tr>
<tr>
<td>c_4 = $10</td>
<td>Slow repair cost</td>
</tr>
<tr>
<td>c_5 = $25</td>
<td>Purchase cost</td>
</tr>
<tr>
<td>c_6 = $0.5</td>
<td>Inventory holding cost</td>
</tr>
<tr>
<td>c_7 = $40</td>
<td>Backorder penalty cost</td>
</tr>
</tbody>
</table>

Table 2: Cost data.

<table>
<thead>
<tr>
<th>Number of items to purchase from external supplier</th>
<th>Number of failed items to enter slow repair service</th>
<th>Number of failed items to enter fast repair service</th>
<th>Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3: Optimal solution for Example 1.

Fig. 5. Optimal flows for sub-network #1.
Following Claim 1, the initial network in Fig. 2 can be decomposed into two sub-networks as presented in Fig. 4, in which we add a cutting line to separate the sub-networks. Recall that the sub-network on the left of the cutting line is called sub-network #1 and the network on the right of the cutting line is called sub-network #2. We apply the proposed algorithm to each sub-network individually. The obtained solutions are presented as the network flows in Figs. 5 and 6, respectively, while the corresponding flow routes generated by the algorithm are given in the Appendix.

In Step S4, the sub-network #1, after constructing the augmenting routes, presented in Table A1 in the Appendix, contains three negative cycles, namely $S-1-2-9-16-20-S$, $S-3-4-9-16-20-S$, and $S-7-8-10-17-16-20-S$, with the costs $0.05 + 15 - 25 = -9.95$, $0.05 + 15 - 25 = -9.95$, and $0.05 + 15 - 0.5 - 25 = -10.45$, respectively. After deleting these cycles, the optimal solution for sub-network #1 is presented in Fig. 5.

The sub-network #2, after constructing the augmenting routes (Table A2 in the Appendix), does not contain any negative cycle. The corresponding optimal solution is presented in Fig. 6. The optimal solution for the original problem is obtained as the union of the two partial solutions in Figs. 5 and 6, as presented in Table 4. The minimum cost obtained is $1703.65, which is lower than the corresponding solution in the non-interval case examined in Example 1.

6. Conclusions

In this paper we study a repairable-item, multi-echelon inventory system with multiple supply alternatives, namely supplies from an external supplier and supplies from repair services. We consider deterministic demand for the spare parts that can be predicted with a predetermined level of accuracy. Over the planning horizon, demand varies over time due to changes in the utilization rate of the parts. Our network flow model makes it possible to find a trade-off between the optimal number of items that need to be repaired at each repair mode and the optimal number of new items that need to be purchased from the external supplier. We prove that the Wagner–Whitin property, known for the lot-sizing problem, can be extended to the spare-part inventory management problem under study. We propose a new minimum-cost network flow algorithm for the case with interval data, which is simple and computationally inexpensive. We present numerical examples to illustrate the advantages of the new model with interval data over a related model with fixed data.

Despite its simplicity, our model is more general than several similar network models in the literature (e.g., Prager, 1956; Ford and Fulkerson, 1962; Zangwill, 1969; Golany et al., 2001; Gass, 2003; Liu et al., 2005; Ahuja and Hochbaum, 2008). The network approach allows us to consider different parties in a supply chain (e.g., bases, depots, transportation facilities, and suppliers) as links in a flow network and include the cost/time aspect of the entire supply chain in the analysis, which, in turn, helps increase collaboration between the parties at the planning stage (Huiskonen, 2001). From the practitioner’s perspective, inventory managers could use our results for the optimal planning and ordering of spare parts in real-life situations where demand is flexible and known up to a certain degree of accuracy. This line of research can be extended by looking at spare-part inventory systems for deteriorating and age-dependent items. Another possible direction for future research is to study non-linear costs and tradeoffs between repair cost, outsourcing cost, backlogging cost, and shortage penalty pervasive in real-life spare-part inventory systems.


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7. Appendix

Consider in more detail the steps of the algorithm involved in solving Example 2. As a result of Step 2.2, we obtain the following routes from $S$ to $T$ and the corresponding flows on these routes in the network $N(s', t')$, which are presented in Fig. 5 and Table A1 for sub-network #1, and in Fig. 6 and Table A2 for sub-network #2.

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51

Table A1
Paths for sub-network #1.

<table>
<thead>
<tr>
<th>Flow value</th>
<th>Path</th>
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<tbody>
<tr>
<td>5</td>
<td>S-1-2-9-16-25-43-44-T</td>
</tr>
<tr>
<td>2</td>
<td>S-3-4-9-16-25-43-44-T</td>
</tr>
<tr>
<td>2</td>
<td>S-3-4-9-16-25-45-46-T</td>
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<tr>
<td>4</td>
<td>S-7-8-10-17-26-49-50-T</td>
</tr>
<tr>
<td>1</td>
<td>S-7-8-10-17-26-47-48-T</td>
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<tr>
<td>6</td>
<td>S-5-6-10-17-26-47-48-T</td>
</tr>
<tr>
<td>4</td>
<td>S-20-16-25-45-46-T</td>
</tr>
<tr>
<td>2</td>
<td>S-20-16-17-26-47-48-T</td>
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Table A2
Paths for sub-network #2.

<table>
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</tr>
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<td>S-13-22-33-34-T</td>
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<td>6</td>
<td>S-18-14-23-37-38-T</td>
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<td>7</td>
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<td>5</td>
<td>S-13-22-31-27-28-T</td>
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<tr>
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</tr>
<tr>
<td>4</td>
<td>S-18-14-23-35-31-32-T</td>
</tr>
<tr>
<td>6</td>
<td>S-18-14-23-35-36-T</td>
</tr>
</tbody>
</table>

51