Journal of Manufacturing Systems 33 (2014) 16–26

Contents lists available at ScienceDirect

Journal of Manufacturing Systems

journal homepage: www.elsevier.com/locate/jmansys

Technical Paper

Dynamic repair priority for a transfer line with a finite buffer

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ARTICLE INFO

Article history:
Received 11 May 2012
Received in revised form 4 October 2013
Accepted 5 November 2013
Available online 17 December 2013

Keywords:
Manufacturing transfer line systems
Algorithms
Queuing theory
Unreliable machines

ABSTRACT

We formulate a continuous-time Markov chain model of a transfer line in which there are two unreliable machines separated by a finite buffer. Due to limited repair resources, simultaneous repairs are not possible in cases where both machines fail, and therefore we develop a repair priority rule that depends on the number of work-pieces present in the buffer. Each machine is characterized by three exponentially distributed random variables: processing time, time to failure, and time to repair. We provide a stochastic model for finding an optimal repair priority rule and an efficient algorithm accompanied by easy-to-use Matlab software. An extensive numerical study is performed to test the sensitivity of the proposed dynamic repair priority rule. While in previous studies repair priority was given to the bottleneck machine, we show that there are situations in which priority should be given to the non-bottleneck machine. Finally, we identify conditions in which adding a second technician is economically advisable.
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1. Introduction

Machines in production systems are often unreliable. Failures can occur in a production line at any given time, disturbing the flow of material through the line and reducing the line production rate. To repair a machine, i.e., restore it to an operational state, it is necessary to allocate resources. Such resources include qualified technicians, of whom there may be a limited number, owing to economic considerations or a shortage of available qualified professionals. To maintain a production line's overall performance in cases of failure, it is necessary to define a repair priority rule that determines how limited repair resources are allocated. Appropriate prioritization of repairs can reduce machines' non-productive time (down time or idle state), as well as reduce the effective recovery time (the time period from machine failure until the machine starts processing work-pieces again).

This paper studies the effect of repair priority on the production rate of a line consisting of two machines separated by a finite buffer. In the model proposed herein, work-pieces enter the first machine, M_1, and an operation takes place. Once processed by M_1, these pieces move on to the buffer, where they stay until taken to the second machine, M_2, for further processing. When processing by M_2 is complete, the work-pieces leave the system (see Fig. 1).

Processing, failure, and repair times for each machine are assumed to be exponential random variables. The buffer capacity is finite, and there is only one technician available to repair both machines. The latter assumption implies that only one machine can be repaired at a time and that when both machines are down, a repair priority has to be established. In this study the priority rule is dependent on a single decision variable—the number of work-pieces in the buffer—while the objective is to maximize the system's production rate.

To obtain an optimal solution, we use a continuous-time discrete-state Markov chain and construct an algorithm that computes the probabilities of various states and the optimal repair priority rule. In addition, we perform a numerical study and sensitivity analysis to examine the influence of the repair priority rule on system performance under various conditions. These analyses show how the proposed dynamic policy outperforms simpler static policies that are not influenced by the state of the system.

Although transfer line modeling has been reviewed extensively [1–4], the literature on the subject of repair priority under resource-constrained conditions is quite limited. The latter can be divided into two streams. The first stream deals with static repair priority, in which the repair priority rule is fixed and is independent of the state of the manufacturing line in the event of failure. Bryant and Murphy [5] developed a model with a static repair priority rule, where the repair priority is given to the slowest machine in the line, independent of the state of the system. Yeralan and Muth [6] compared between two scenarios in which either the first or the second machine has repair priority. If two or more machines fail simultaneously, a predetermined and unchanging repair policy dictates which machine is the first to be repaired. Dogan-Sahiner

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http://dx.doi.org/10.1016/j.jmsy.2013.11.001

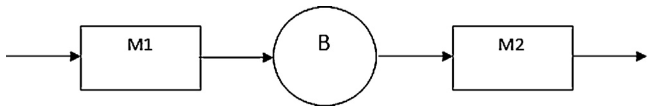

Fig. 1. Two-machine transfer line.

and Altiok [7] allowed simultaneous repairs, taking into account the sum of all individual repair rates at any given time as a constraint.

Dudick [8] was the first to model 'dynamic' repair priority, in which the repair priority rule is based on the state of the system. In Dudick's model, when two machines in a line are down simultaneously, the repair priority is determined according to the number of pieces in the buffer. Dudick assumes a discrete production line comprising two machines, in which the processing time for each machine is fixed and equal to one unit of time. The failure times in his model are geometrically distributed, and repair times are either constant or geometrically distributed. Buzacott [9] considers similar assumptions, but his model, unlike Dudick's, dictates that if a machine breaks after the repair of another machine has already begun, the original repair must be completed without interruption. Rho [10] developed a dynamic repair priority rule for a transfer line with identical machines; each machine is served by a robot that feeds pieces into the machine and removes pieces from it. Yeralan and Dieck [11] developed a dynamic repair priority system in which the repair rates change as a function of the number of pieces in the buffer. Their model assumes that the technician will work at a faster rate if necessary. While the above papers each considered a discrete production system comprising two machines with identical processing times, in the current paper we develop a dynamic repair priority rule for a continuous production system in which each machine is different and characterized by three exponential random variables: processing, repair and failure times.

Models of long transfer lines, consisting of more than two machines, require solutions of much greater complexity owing to the large state space, and they are generally investigated using either simulation-based or analytical methods. Smith [12] and Um et al. [13] provide detailed surveys on the use of simulation for the design and operation of manufacturing systems. Yang et al. [14] propose an original analytic method, a new parameter coupling method, and compare their analytical results to the results of a simulation experiment. The latter study focuses on the context of a closed-loop manufacturing system (CLMS). A two-node CLMS is described in our paper in Section 4. Simulation-based studies of long transfer lines that consider resource constraints (a single technician) include those of Smith [15], Kouikoglou and Phillis [16], and Chakravarthy and Agarwal [17]. Studies involving analytic methods include those of Gershwin [18], Alvarez-Vargas et al. [19], Tan and Karabati [20], Kouikoglou [21], Kim and Gershwin [22], Kuhn [23] and Xia et al. [24]. Among these, Kuhn [23] is the only study to consider repair resource constraints. Kuhn [23] determined the production rate of the transfer line by using two coupled queuing systems. Kuhn's model assumes that when multiple machines are awaiting repair, they are serviced in first-come-first-serve order. In this paper, we identify an optimal dynamic repair priority rule that depends on the number of items in the buffer, and assume that the technician immediately services the highest priority machine, even if it breaks down while he is repairing a lower-priority machine.

The main contributions of our paper are a Markov chain model for implementing a dynamic repair priority policy, and a corresponding solution technique to find the optimal priority rule. The proposed model can also serve as a tool to assist operation managers in deciding whether it is economical to add an extra technician.

2. Model assumptions and description

We study a system consisting of two machines and one buffer located between them, as described in Fig. 1. Work-pieces enter M_1 from an outside source such as a raw material inventory. Finished work-pieces from M_1 are transferred to the buffer (denoted by B in Fig. 1), where they wait until M_2 is available to further process them. Each machine processes one work-piece at a time. The buffer has a limited capacity of pieces, denoted by the parameter N. The buffer capacity includes the work-piece in M_2. Work-pieces from M_2 are transferred onwards once finished. The state of machine M_i is denoted by α_i. When α_i is 1 the machine is "up", and when α_i is 0 the machine is "down". In this study, a "down" state means that the machine is not operational, cannot process any work-pieces, and is either waiting for repair or under repair. The situation in which M_1 is "up" and the buffer is full is called *blockage*. M_1 will start processing the next work-piece only after an empty space becomes available in the buffer. When M_2 is "up" but the buffer is empty, M_2 has no work-piece to process and therefore remains idle. This situation is called *starvation*. A machine can only fail during processing, and therefore a machine's state cannot be changed from "up" to "down" if the machine is in a blockage or a starvation situation. It is assumed that there will always be an available work-piece for M_1 to work on and available space to transfer a complete work-piece from M_2.

The state of the system is denoted by $S = (n, \alpha_1, \alpha_2)$, where n is the number of work-pieces waiting in the buffer. The steady-state probability that the system will be in a certain state is denoted by $p(n, \alpha_1, \alpha_2)$. Each machine is characterized by three exponential random variables: the processing time (with mean $1/\mu_i$), the time to repair (with mean $1/r_i$, abbreviated MTTR) and the time to failure (with mean $1/p_i$, abbreviated MTTF).

We assume that there is only one technician, and in the case when both machines are down a preemptive repair discipline is followed. That is if a machine with a higher priority fails when the technician repairs the other machine, the technician interrupts the current repair and immediately starts repairing the machine of high priority.

In our model the repair priority is determined by one decision variable, L, where $1 \le L \le N$. When the number of pieces in the buffer is equal to or greater than L, M_2 is repaired first; otherwise, M_1 is repaired first. Our objective is to find the optimal L that maximizes the line production rate, P, which is the number of work-pieces produced in a given period of time.

To compute the line production rate we first calculate each machine's *efficiency* E_i defined as the fraction of time during which machine i produces pieces. We can express E_1 and E_2 as follows:

$$E_1 = \sum_{n=0}^{N-1} \sum_{\alpha_2=0}^{1} p(n, 1, \alpha_2) \tag{1}$$

$$E_2 = \sum_{n=1}^{N} \sum_{\alpha_1=0}^{1} p(n, \alpha_1, 1) \tag{2}$$

The quantity $\mu_i E_i$ is the production rate of M_i, i.e., the rate at which pieces emerge from machine i. According to the conservation of flow (see, for example, Lemma 5 in [25]) the rate at which pieces emerge from M_1 is equal to the rate at which they emerge from M_2. The line production rate, P, defined as the rate at which work-pieces emerge from the production line, is thus:

$$P = \mu_1 E_1 = \mu_2 E_2 \tag{3}$$

Table 1a
Balance equations for the case of $L_{opt} = (N+1)/2$.

State		Balance equations
α_1	α_2	Boundary states for an empty buffer ($n=0$)
0	0	$p(0,0,0)\times r_1 = p(0,1,0)\times p_1$
0	1	$p(0,0,1)\times r_1 = p(1,0,1)\times \mu_2 + p(0,1,1)\times p_1$
1	0	$p(0,1,0)\times(\mu_1+p_1+r_2) = p(0,0,0)\times r_1$
1	1	$p(0,1,1)\times(\mu_1+p_1) = p(1,1,1)\times\mu_2 + p(0,0,1)\times r_1 + p(0,1,0)\times r_2$
		Internal states for $1\le n\le N-1$
0	0	$p(n,0,0)\times r_1 = p(n,0,1)\times p_2 + p(n,1,0)\times p_1$
0	1	$p(n,0,1)\times(\mu_2+r_1+p_2) = p(n,1,1)\times p_1 + p(n+1,0,1)\times\mu_2$
1	0	$p(n,1,0)\times(p_1+r_2+\mu_1) = p(n-1,1,0)\times\mu_1 + p(1,0,0)\times r_1 + p(n,1,1)\times p_2$
1	1	$p(n,1,1)\times(\mu_1+\mu_2+p_2+p_1) = p(n-1,1,1)\times\mu_1 + p(n+1,1,1)\times\mu_2 + p(n,0,1)\times r_1 + p(n,1,0)\times r_2$
		Boundary states for a full buffer ($n=N$)
0	0	$p(N,0,0)\times r_1 = p(N,0,1)\times p_2$
0	1	$p(N,0,1)\times(\mu_2+r_1+p_2) = 0$
1	0	$p(N,1,0)\times r_2 = p(N-1,1,0)\times\mu_1 + p(N,1,1)\times p_2$
1	1	$p(N,1,1)\times(\mu_2+p_2) = p(N-1,1,1)\times\mu_1 + p(N,0,1)\times r_1 + p(N,1,0)\times r_2$

We also consider each machine's *efficiency*, e_i defined as the fraction of time machine i would be producing pieces if there were no blockage or starvation (see Eq. (3) in [23]):

$$e_i = \frac{MTTF_i}{MTTF_i + MTTR_i + w_i} \tag{4}$$

where w_i is the mean waiting times for a technician for machine i. As described in Section 2, when $n < L$, repair priority is given to M_1, so the waiting time for M_1 is zero. When L increases w_1 decreases and w_2 increases therefore the following lemma holds:

Lemma 1. *The efficiency of M_1 is an increasing function of L, and the efficiency of M_2 is a decreasing function of L.*

The isolated production rate is therefore the rate at which machine i would process parts in isolation, given in:

$$\mu_i \frac{r_i}{r_i + p_i} \tag{5}$$

3. Balance equations

In this section, to obtain the steady-state probability for each state of the system, we construct Markov process balance equations that each equate the rate of leaving a given state with the rate of entering it for a particular value of L. Specifically, Table 1a presents the balance equations for $n < L$, when the repair priority is given to M_1. Table 1b presents the balance equations for $n \ge L$, when the repair priority is given to M_2.

Each equation in Tables 1a and 1b refers to the state noted in the two columns to its left. The left side of the equation represents the rate of leaving that state, and the right part of the equation represents the rate of entering that state. According to the above, there are $4\times(N+1)$ equations to solve, each equation relating to a different possible state of the system (four different combinations of machines' "up" and "down" states, times $N+1$ different possible quantities of items in the buffer, including zero). The assumptions of the model imply that certain boundary states are transient, that is, their steady-state probability is zero. In particular, according to Lemma 1 in [25], the states (0, 0, 0), (0, 1, 0), (N, 0, 0), and (N, 0, 1) are transient. Thus there are four boundary equations which are identical for the two cases (Tables 1a and 1b).

4. Optimal repair priority

In order to find the value of L that yields the optimal repair priority (i.e., that maximizes the line production rate), we consider the following two-node closed Jackson network, where each node represents one machine, denoted by M_i. The network is described in Fig. 2.

The description of this network is similar to that of the transfer-line system in Section 2. The state of M_i is represented by α_i, such that when α_i is 1 the machine is "up", and when α_i is 0 the machine is "down". Each machine is characterized by three exponential random variables as in the original system. The number of work-pieces in the closed network is equal to N, which is the size of the buffer of the transfer-line system. The flow of work-pieces in the network is as follows: A work-piece enters the first machine's (M_1's) queue, waits until M_1 is available, is processed by M_1, continues to M_2's queue, gets processed by M_2, continues to M_1, and so on.

Table 1b
Balance equations for the case of $n \ge L$.

State		Balance equations
α_1	α_2	Boundary states for an empty buffer ($n=0$)
0	0	$p(0,0,0)\times r_2 = p(0,1,0)\times p_1$
0	1	$p(0,0,1)\times r_1 = p(1,0,1)\times\mu_2 + p(0,1,1)\times p_1 + p(0,0,0)\times r_2$
1	0	$p(0,1,0)\times(\mu_1+p_1+r_2) = 0$
1	1	$p(0,1,1)\times(\mu_1+p_1) = p(1,1,1)\times\mu_2 + p(0,0,1)\times r_1 + p(0,1,0)\times r_2$
		Internal states for $1\le n\le N-1$
0	0	$p(n,0,0)\times r_2 = p(n,0,1)\times p_2 + p(n,1,0)\times p_1$
0	1	$p(n,0,1)\times(\mu_2+r_1+p_2) = p(n,1,1)\times p_1 + p(n+1,0,1)\times\mu_2 + p(n,0,0)\times r_2$
1	0	$p(n,1,0)\times(p_1+r_2+\mu_1) = p(n-1,1,0)\times\mu_1 + p(n,1,1)\times p_2$
1	1	$p(n,1,1)\times(\mu_1+\mu_2+p_2+p_1) = p(n-1,1,1)\times\mu_1 + p(n+1,1,1)\times\mu_2 + p(n,0,1)\times r_1 + p(n,1,0)\times r_2$
		Boundary states for a full buffer ($n=N$)
0	0	$p(N,0,0)\times r_2 = p(N,0,1)\times p_2$
0	1	$p(N,0,1)\times(\mu_2+r_1+p_2) = p(N,0,0)\times r_2$
1	0	$p(N,1,0)\times r_2 = p(N-1,1,0)\times\mu_1 + p(N,1,1)\times p_2$
1	1	$p(N,1,1)\times(\mu_2+p_2) = p(N-1,1,1)\times\mu_1 + p(N,0,1)\times r_1 + p(N,1,0)\times r_2$

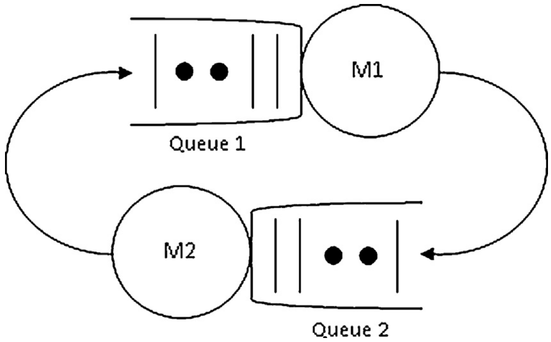

Fig. 2. Two-node closed Jackson network.

The production rate of this network is the number of work-pieces that complete one cycle in a unit of time. The queue length for machine M_i, including the work-piece being processed, is denoted by n_i, $n_1+n_2=N$. We assume there is only one technician, so if both machines are down, the repair priority is determined by one decision variable, L, where $1 \le L \le N$. When $n_2 \ge L$, M_2 is repaired first; otherwise, M_1 is repaired first.

Proposition 1. *The transfer-line system and the closed Jackson network are equivalent, i.e., they have identical steady state probabilities.*

Proof. Denote the transfer-line system and the Jackson network as Model 1 and Model 2, respectively. The machines in Model 1 and Model 2 have the same attributes. The blockage situation in Model 1 is equivalent to the situation in Model 2 where $n_2=N$. The starvation situation in Model 1 is equivalent to the situation in Model 2 where $n_1=N$. The situation in Model 1 where the buffer contains n units $(1 \le n \le N-1)$ is equivalent to the situation in Model 2 where $n_2=n$ and $n_1=N-n$. Since all the states and attributes of Model 1 are identical to those of Model 2, the models are equivalent. □

Proposition 2. *If $L=x$ yields the optimal repair policy for a transfer line where the first machine is characterized by: μ_1, r_1 and p_1, and the second machine is characterized by: μ_2, r_2 and p_2, then $L=N+1-x$ yields the optimal repair policy for a transfer line where the first machine is characterized by: μ_2, r_2, and p_2, and the second machine is characterized by: μ_1, r_1 and p_1.*

Proof. Let $L=x$ be the optimal repair policy in the Jackson network. It means that if the number of units in queue for machine M_2, $n_2 \ge x$ then M_2 is repaired first. Since $n_1+n_2=N$ this condition is equivalent to $n_1<N+1-x$. Now switching the machines' places the claim is proved. □

Therefore, for a transfer line with identical machines ($r_1=r_2, p_1=p_2, \mu_1=\mu_2$), the optimal repair priority is obtained for $L_{opt}=(N+1)/2$. In other words, when both machines are identical, the optimal repair priority rule is to repair M_2 before M_1 if the buffer is at least half full.

Proposition 3. *When the two machines are identical, the system production rate is a symmetric function of L.*

Proof. Let $P(x)$ be the line production rate in the Jackson network when $L=x$ that is the number of work pieces that completes one cycle of being processed by M_1 and by M_2 in a unit of time. Since both machines are identical this is the same rate as the number of work pieces that completes one cycle of being processed by M_2 and by M_1 when $L=N+1-x$. Thus $P(N+1-x)=P(x)$. □

In cases in which the machines are not identical, the following conjecture, although not proven analytically, is supported by numerous numerical examples (see Section 6):

Conjecture 1. *The production rate is a unimodal function of L.*

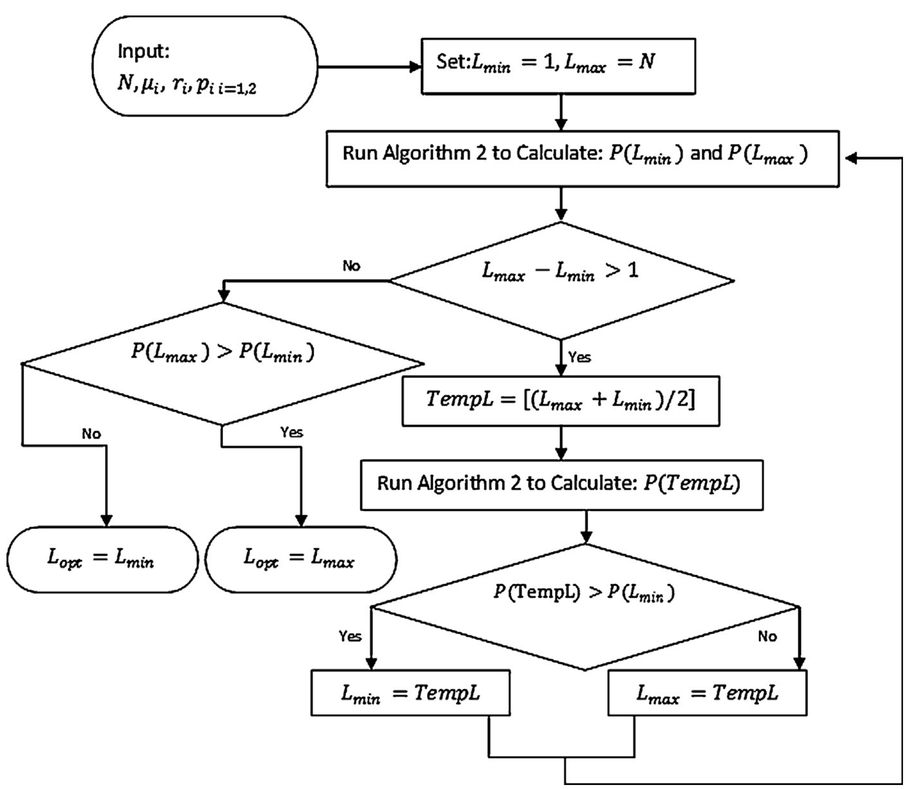

Fig. 3. Algorithm 1: The optimal repair priority *L* value search.

Algorithm 2: The system production rate for a given L

1. Input: buffer size and machine characteristics: $N, \mu_i, r_i, p_{i\ i=1,2}$
2. Input: L
3. Output: Production rate, denoted by P(L)
4. Calculate $A, B, C, D, E,$ and F according to *Table 3* and *Table 4* (For the given L)
5. $T \leftarrow r_1 A + r_2 B + p_1 C + p_2 D + \mu_1 E + \mu_2 F$
6. Find all steady state probabilities
7. $E_1 \leftarrow \sum_{n=0}^{N-1} \sum_{\alpha_2=0}^{1} p(n, 1, \alpha_2)$
8. $P(L) \leftarrow E_1 \mu 1$
9. *Return* $P(L)$

Fig. 4. Algorithm 2: The system production rate for a given L.

5. Solution technique

To obtain L_{opt}, the value of L that maximizes the line production rate, we use Algorithm 1, in which a binary search is carried out on the interval [1, N] (see Fig. 3). Such a search is possible under the assumption that the line production rate is a unimodal function of L (Conjecture 1).

Algorithm 2 (see Fig. 4), a sub-procedure in Algorithm 1, calculates the line production rate P for a given value of L in the following two steps: (1) find the steady-state probabilities for all states in which M_1 is up. (2) Multiply the probability that M_1 is working (by summing up the probabilities obtained in step 1) by μ_1. Denote by T the coefficient matrix representing all balance equations from Tables 1a and 1b. T is a sum of six matrices (A, B, C, D, E and F), each multiplied by one of the parameters of the production line:

$$T = r_1 A + r_2 B + p_1 C + p_2 D + \mu_1 E + \mu_2 F \tag{6}$$

We have determined the structure of each matrix by solving numerical examples and observing the pattern of each coefficient. We then formulated these patterns into logical terms to obtain a description of the general case. The logical terms constituting each matrix element are described in Tables 2 and 3. The notation A_{ij} denotes the value in matrix A, in line i, column j, where α_1, α_2, and n are j-dependent as described in Table 3.

Table 2
Transition matrix elements.

Term	$n < L$	$n \geq L$
A_{ij}	1 if $\alpha_1(i,j) = 0$ and $i = j$ -1 if $\alpha_1(i,j) = 0$ and $i = j+2$ else, 0	1 if $\alpha_1 < \alpha_2$ and $i = j$ -1 if $\alpha_1 < \alpha_2$ and $i = j+2$ else, 0
B_{ij}	1 if $\alpha_1 > \alpha_2$ and $i = j$ -1 if $\alpha_1 > \alpha_2$ and $i = j+1$ else, 0	1 if $\alpha_2(i,j) = 0$ and $i = j$ -1 if $\alpha_2(i,j) = 0$ and $i = j+1$ else, 0
C_{ij}	1 if $n < N$, $\alpha_1 = 1$ and $i = j$ -1 if $n < N$, $\alpha_1 = 1$ and $i = j-2$ else, 0	
D_{ij}	1 if $n > 0$, $\alpha_2 = 1$ and $i = j$ -1 if $n > 0$, $\alpha_2 = 1$ and $i = j-1$ else, 0	
E_{ij}	1 if $n < N$, $\alpha_1 = 1$ and $i = j$ -1 if $n < N$, $\alpha_1 = 1$ and $i = j+4$ else, 0	
F_{ij}	1 if $n > N$, $\alpha_2 = 1$ and $i = j$ -1 if $n > N$, $\alpha_2 = 1$ and $i = j-1$ else, 0	

Proposition 4. *The complexity of Algorithm 1 is* $O(N^3 \times \log N)$.

Proof. The line production rate for a given value of L is calculated by solving a $4 \times (N+1)$ matrix of linear equations. This step (see line 6 in Algorithm 2) is done in $O\{[4 \times (N+1)]^3\} = O(N^3)$ time [26]. A binary search is performed on the interval [1, N]; therefore, the total complexity of Algorithm 1 is $O(N^3 \times \log N)$. □

The implementation of Algorithm 1 can be downloaded from [28].

6. Sensitivity analysis

In this section we investigate how modifying each production line parameter affects the optimal value of L (L_{opt}) and the line production rate (P). We start by varying the value of either the processing rate, failure rate or repair rate while maintaining the other variables constant. Table 4 presents three different analyzed cases. Next, we investigate how the buffer capacity affects L_{opt} and P.

6.1. Case 1

In the first case we analyze a production line in which the two machines have identical repair rates $r_1 = r_2$ and identical failure rates $p_1 = p_2$ but different processing rates μ_1, μ_2. We observe the effect of L on the line production rate, while varying the difference between the machines' processing rates, $\Delta\mu = \mu_2 - \mu_1$. For simplicity, the lower of the two processing rates is always assigned the same value, $\min(\mu_1, \mu_2) = 5$. Thus, if $\mu_1 > \mu_2$ then $\mu_2 = 5$, and if

Table 3
System state variable value for each element of the transition matrices.

Variable	Value
$\alpha_1(i,j)$	If $j = 1+4i$ or $j = 2+4i$ then $\alpha_1(i,j) = 0$ else, $\alpha_1(i,j) = 1$ where i is an integer $i = 0, 1, 2, \ldots, 4(N+1)$
$\alpha_2(i,j)$	if $j = 1+2i$ then $\alpha_2(i,j) = 0$ else, $\alpha_2(i,j) = 1$ where i is an integer $i = 0, 1, 2, \ldots, 4(N+1)$
$n(i,j)$	if $i < \frac{j-1}{4} < i+1$ then $n(i,j) = i$ where i is an integer $i = 0, 1, 2, \ldots, 4(N+1)$

Table 4
Summary of three numerically studied cases.

Case	Varied parameters	Range	Graphs
1	$\Delta\mu = \mu_2 - \mu_1$ L	(−15) to 15 1–21	Figs. 4 and 5
2	$\Delta p = p_2 - p_1$ L	(−5) to 5 1–21	Figs. 6 and 7
3	$\Delta r = r_2 - r_1$ L	(−9) to 9 1–21	Figs. 8 and 9

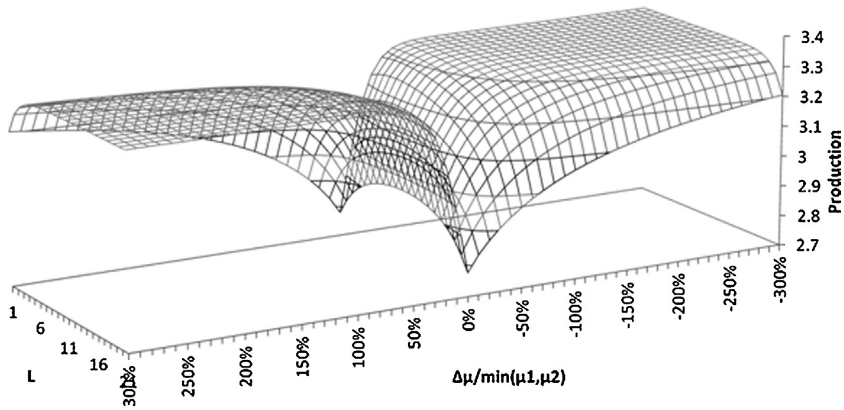

Fig. 5. The line production rate P as a function of L and the difference in the machines' processing rates $\Delta\mu = \mu_2 - \mu_1$, while $\min(\mu_1, \mu_2) = 5$.

$\mu_1 < \mu_2$ then $\mu_1 = 5$. The following graphs present results for a specific production line characterized by: $r_1 = r_2 = 10$, $p_1 = p_2 = 5$, $N = 21$ and $\min(\mu_1, \mu_2) = 5$.

Fig. 5 presents the line production rate, which has a twofold symmetry property as a function of $\Delta\mu/\min(\mu_1, \mu_2)$ and L. The optimal L value for a given $\Delta\mu$ is marked in stars in Fig. 6. Due to the symmetry property noted above, Fig. 6 describes only cases where $\mu_2 > \mu_1$. When both machines are identical, i.e., $\Delta\mu = 0$, the value of $L_{opt} = (N + 1)/2 = 11$ (Proposition 2).

The line production rate and L_{opt} increase when μ_1 remains equal to 5 and μ_2 is increased (Fig. 6). When the processing rate of M_2 is higher than that of M_1, it is advantageous to assign priority to M_2 only in states where the buffer is sufficiently full. The probability that M_2 will cause system blockage decreases when μ_2 increases, and L_{opt} increases accordingly. In addition, it can be seen (Fig. 6) that the optimal priority rule dominates static repair rules such as always fixing M_2 (L=1) or always fixing M_1 (L=21). Thus, implementing a static priority rule such as first repairing the bottleneck machine (i.e., the machine with the slower isolated production rate) is not the best policy in terms of optimizing the line production rate. This can be seen in the following cases as well.

6.2. Case 2

In the second case we maintain the machines' repair rates and processing rates at constant, equal values and vary the failure rates, p_1, p_2. The following graphs present results for a production line characterized by: $r_1 = r_2 = 5$, $\mu_1 = \mu_2 = 5$, $N = 21$ and $\min(p_1, p_2) = 5$.

Fig. 7 presents the line production rate as a function of Δp and L in the case of machines that are identical in terms of failure rates and repair rates, where the minimum failure rate is held at a constant value. As in Case 1, the line production rate function has a twofold symmetry property.

As the failure rate of M_2 increases, the line production rate decreases, and L_{opt} decreases as well (see Fig. 8). As the difference in the failure rates of the machines, Δp, increases, it is favorable to repair M_1 first if the empty part of the buffer will not be filled before M_2 is repaired. The decreasing value L_{opt} as the failure rate of M_2 increases can be explained by the scenario in which M_2 is repaired after M_1 but experiences a very short "up" time and fails again. In this scenario, M_1 will most probably reach blockage, and in order to prevent that, it is favorable to repair M_1 first only if there is enough empty space in the buffer for it to fill as M_2 experiences multiple failure states.

6.3. Case 3

In this case we examine a production line in which the two machines have identical failure rates $p_1 = p_2$ and processing rates $\mu_1 = \mu_2$ but different repair rates, r_1, r_2. The following graphs present the results for a production line that is characterized by: $p_1 = p_2 = 1$, $\mu_1 = \mu_2 = 5$, $N = 21$ and $\min(r_1, r_2) = 1$.

Fig. 9 presents a twofold symmetry graph, similar to the graphs presented in Cases 1 and 2. As the difference in the repair rates of the machines, Δr, increases, the production rate becomes less stable, and the machine with the slower repair rate affects the line production similarly to a machine with a slower processing rate;

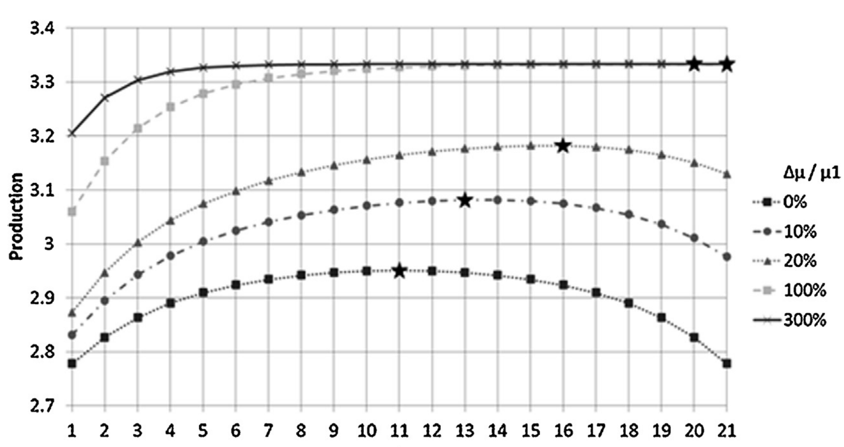

Fig. 6. A few cross sections of the line production rate function described in Fig. 5. Stars mark the maximum line production rate in each case.

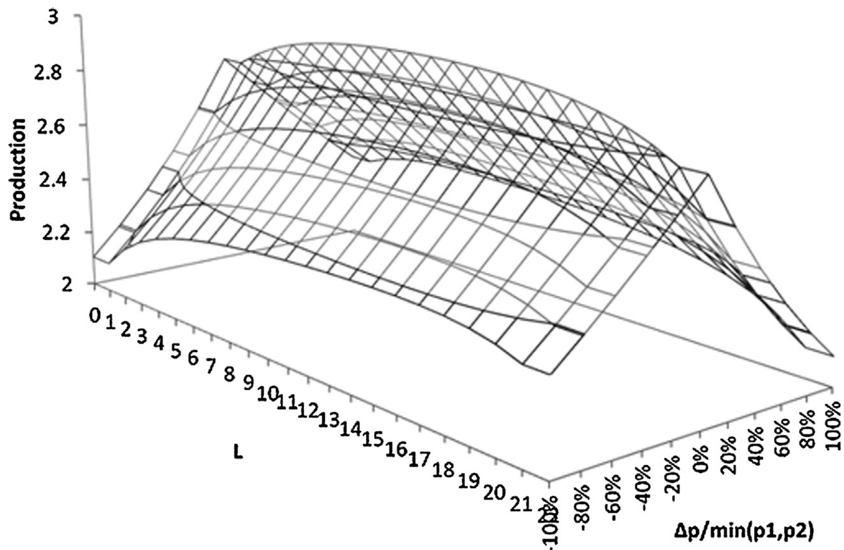

Fig. 7. The line production rate P as a function of L and the difference in machine failure rates $\Delta p = p_2 - p_1$ while $\min(p_1, p_2) = 5$.

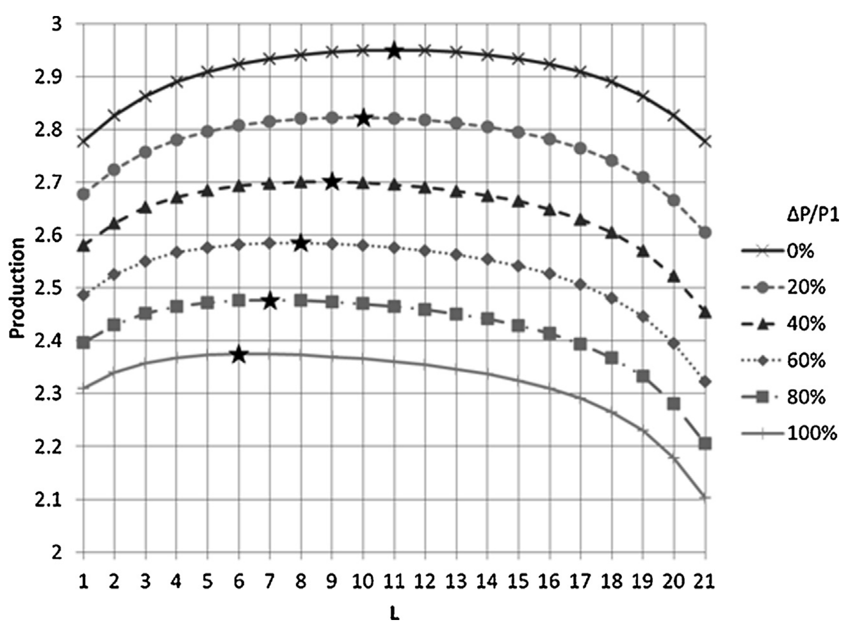

Fig. 8. A few cross sections of the line production rate function described in Fig. 7. Stars mark the maximum line production rate value in each case.

thus, the observations obtained here are similar to those discussed in Case 1. Specifically, from Fig. 10 we observe that when M_2 is repaired faster than M_1, it is preferable to repair M_2 first only if the buffer is more than half full.

Table 5 summarizes the main observations from the three cases.

6.4. Case 4

In this case we investigate how modifying buffer capacity (N) affects L_{opt}. Table 6 presents the parameters of the seven production lines used in this analysis. For each production line, the parameters r, p and μ were maintained constant, and N was varied in the range 5. . .500.

Fig. 11 presents L_{opt} for the entire range of buffer capacity values (5. . .500). In order to analyze whether the sensitivity of L_{opt} to changes in line parameters differs between low values of buffer capacity (e.g., N = 5) and high values of buffer capacity (e.g., N = 500), Fig. 12 zooms in on the values of L_{opt} for the range N = 5. . .N = 50.

Table 5
Summary of main observations.

Case	Condition	L_{opt} value	Comments
1	$\mu_1 > \mu_2$	$1 < L_{opt} < (N+1)/2$	For $\mu_1 \gg \mu_2$ $L_{opt} \to 1$
	$\mu_1 = \mu_2$	$L_{opt} = (N+1)/2$	Identical machines
	$\mu_1 < \mu_2$	$(N+1)/2 < L_{opt} < N$	For $\mu_1 \ll \mu_2$ $L_{opt} \to N$
2	$p_1 > p_2$	$(N+1)/2 < L_{opt} < N$	For $p_1 \gg p_2$ $L_{opt} \to N$
	$p_1 = p_2$	$L_{opt} = (N+1)/2$	Identical machines
	$p_1 < p_2$	$1 < L_{opt} < (N+1)/2$	For $p_1 \ll p_2$ $L_{opt} \to 1$
3	$r_1 > r_2$	$1 < L_{opt} < (N+1)/2$	For $r_1 \gg r_2$ $L_{opt} \to 1$
	$r_1 = r_2$	$L_{opt} = (N+1)/2$	Identical machines
	$r_1 < r_2$	$(N+1)/2 < L_{opt} < N$	For $r_1 \ll r_2$ $L_{opt} \to N$

All parameters μ_i, p_i, r_i for i = 1, 2 are equal unless noted differently.

For small buffer capacities, L_{opt} is indifferent to small changes in processing rate (Fig. 12). For example, when the buffer capacity is 5 units, L_{opt} = 3 for all line configurations. On the other hand, for higher buffer capacities, each line has a different optimal L

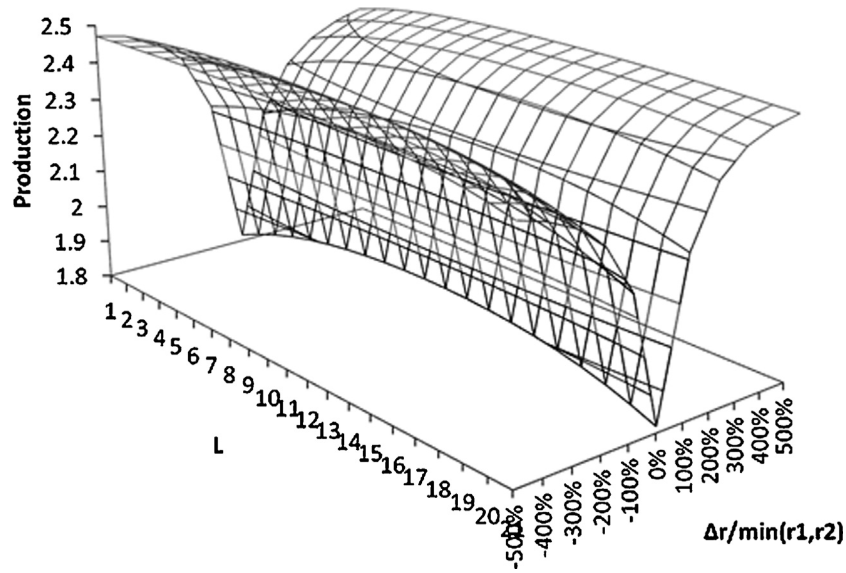

Fig. 9. The line production rate P as a function of L and the difference in the machines' repair rates, $\Delta r = r_2 - r_1$, while $\min(r_1, r_2) = 1$.

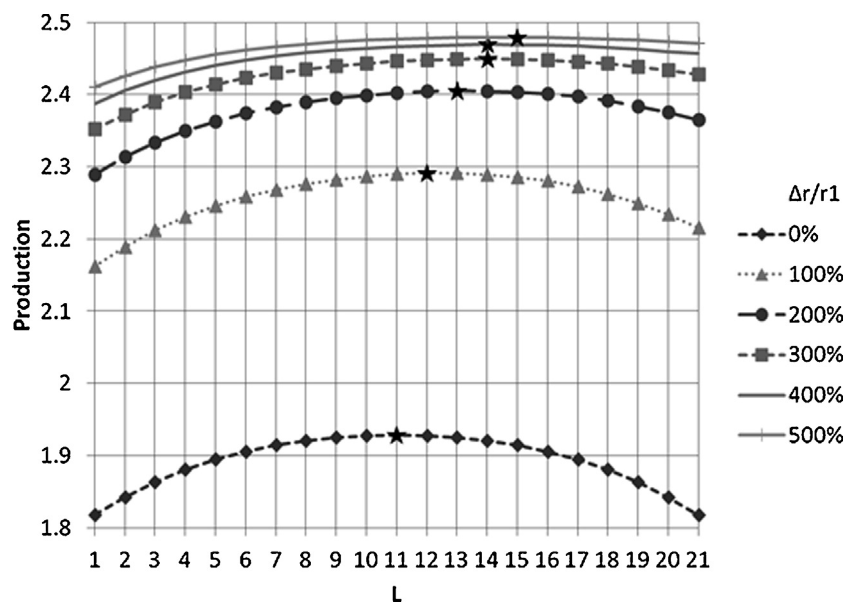

Fig. 10. A few cross sections of the line production rate function described in Fig. 9. Stars mark the maximum line production rate in each case.

Table 6
Calculation input details.

Line	r_1	r_2	p_1	p_2	μ_1	μ_2	N
1	1	1	0.1	0.1	5	5.5	5, 10, 50, 100, 250, 500
2					5	5.25	
3					5	5.05	
4					5	5	
5					5.05	5	
6					5.25	5	
7					5.5	5	

value (Fig. 11). For line 4, in which the two machines are identical, L_{opt} is a linear function of N, in accordance with Proposition 2. For line 1 and line 2, in which the processing rate of M_2 is higher than that of M_1, L_{opt} increases and is getting very close to N (see Case 1). An opposite result is demonstrated in line 6, in which $\mu_1 > \mu_2$; i.e., L_{opt} is reduced. Also note that for line 6, L_{opt} remains the same ($L_{opt} = 8$) regardless of buffer size, since there is maintain the number of pieces in the buffer at a relatively low level in order to reduce the likelihood of blockage.

Fig. 13 presents the line production rate as a function of the buffer capacity under the optimal repair priority rule. Focusing on the lines in which $\mu_2 \geq \mu_1$, we observe that as buffer size increases, the probabilities for blockage and for starvation decrease, and the general production rate increases. Clearly, in these cases

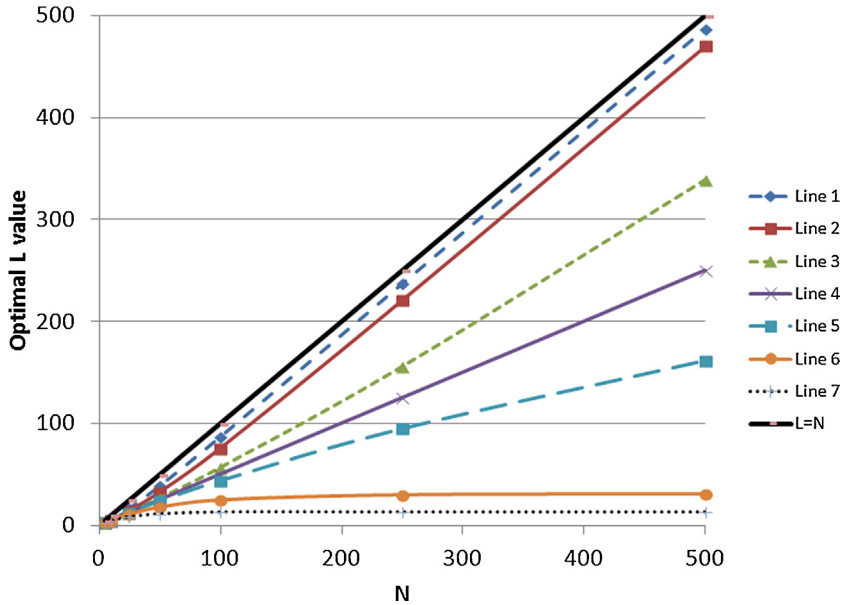

Fig. 11. L_{opt} as a function of buffer capacity N.

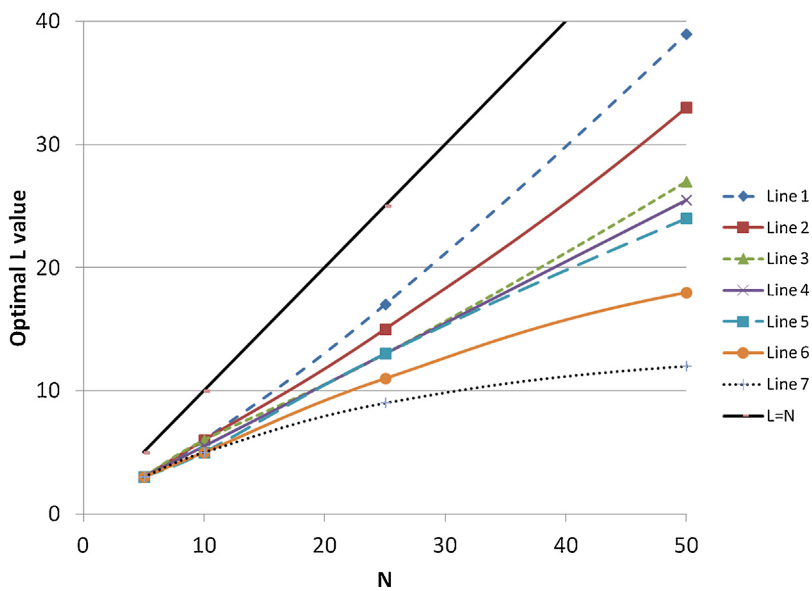

Fig. 12. Close-up of the lower-value area of the graph in Fig. 11.

increasing the buffer size has a diminishing marginal effect. In addition, beyond a certain buffer size the line production rate converges to the isolated production rate of M_1 (see Eq. (5)), because (for cases in which $\mu_2 \geq \mu_1$ and all other parameters are identical for the two machines) a very large buffer capacity approximates a situation in which M_1 is working in isolation.

7. Economic analysis of adding a second technician

To provide a means of analyzing whether it is worthwhile to add another technician, we compare the line production rate under two policies: only one technician who works according to our proposed repair priority rule, denoted by L_{opt}, versus two technicians, i.e., a case in which there is no repair resource constraint. In the following we analyze the case of a production line characterized by: $r_1 = r_2 = 1$, $p_1 = p_2 = 0.1$, $\mu_1 = \mu_2 = 5$ and $N = 21$. We increase one of the parameters of M_2 and plot the difference between the line production rates obtained with the two policies.

As can be seen in Fig. 14, when the difference between μ_1 and μ_2 is large ($\Delta\mu/\mu_1 \geq 100\%$), the presence of an additional technician has almost no effect on the line production rate. The line plot in Fig. 14 shows the additional production rate gained by adding another technician. By multiplying this gap by the profit per unit produced, the actual profit gained from the addition of the technician can be derived. A given technician fee per time unit sets a breakeven point above which hiring an additional technician becomes economically worthwhile.

Similar comparisons are shown in Fig. 15 for machines with different failure rates (and otherwise identical parameters) and in Fig. 16 for machines with different repair rates (and otherwise identical parameters).

The line plot in Fig. 15 shows that the gain in production rate achieved by adding a second technician increases with $\Delta p/p_1$ for

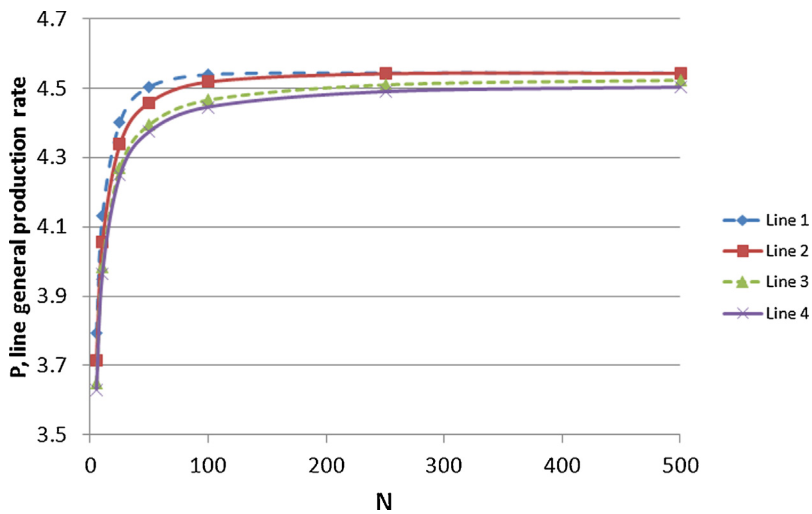

Fig. 13. Line production rate as a function of buffer capacity.

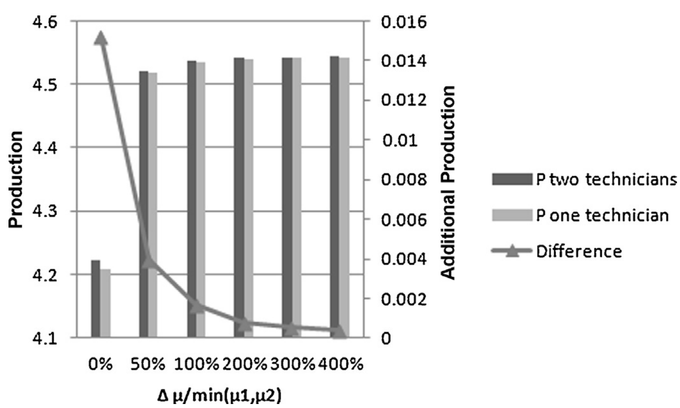

Fig. 14. Line production rate as a function of $\Delta\mu/\mu_1$: Comparison between one and two technicians.

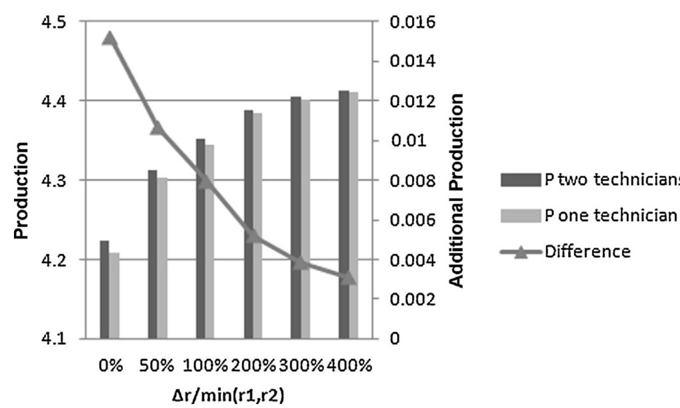

Fig. 16. Line production rate as a function of $\Delta r/r_1$: Comparison between one and two technicians.

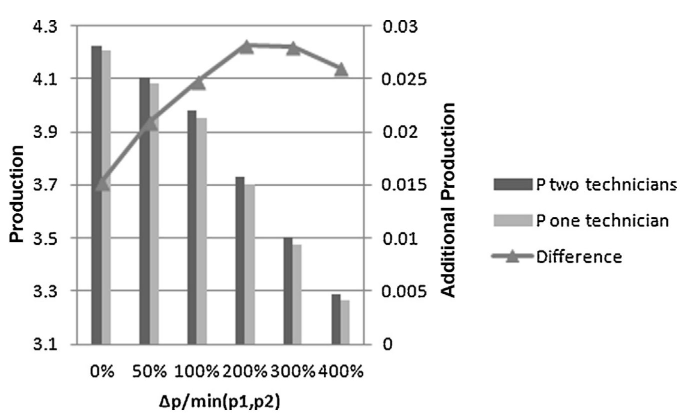

Fig. 15. Line production rate as a function of $\Delta p/p_1$: Comparison between one and two technicians.

$\Delta p/p_1 \leq 200\%$, and after this point the gap decreases but stays positive. The existence of a maximum point can be explained by the fact that adding an extra technician is advantageous only if both machines can begin to produce immediately upon being repaired, without experiencing blockage or starvation. In the case examined, as the failure rate of M_2 increases to high levels, M_1 will experience more blockage states, stay idle and not contribute to the line production rate. As this situation becomes extreme, repairing M_1 immediately after it breaks down or waiting until the repair of M_2 is complete does not make a substantial difference in terms of the production rate; thus, the benefit of adding a technician decreases when $\Delta p/p_1 > 200\%$.

In Fig. 16 we observe that as $\Delta r/r_1$ increases, the gap between the production rates decreases. Note that this decrease is less steep than the decrease observed in Fig. 10. Adding a second technician becomes less economical for very large values of $\Delta r/r_1$.

8. Conclusions

In this paper we constructed a continuous-time Markov process model for a production line consisting of two unreliable machines and one finite buffer. The time to failure, the time to repair and the processing time of each of the machines were exponentially distributed. We introduced an exact numeric solution technique for deriving a dynamic repair priority rule for the production line, based on the number of items present in the buffer. Numeric results were presented and analyzed.

In addition, for cases in which both machines were down, we compared line production rates achieved when a dynamic repair priority rule was applied versus those achieved under a static repair priority policy, wherein priority was given to a specific machine independent of the system state (number of items in the buffer). Main observations are as follows:

- Prioritizing repair of the bottleneck machine, that is, the machine with the lower isolated production rate, is not necessarily the most efficient approach in terms of maximizing the line production rate. This work shows that in certain conditions, prioritizing the non-bottleneck machine can yield a higher line production rate.

- Use of a dynamic repair priority rule that prioritizes a machine's repair according to the quantity of items in the buffer yields the same or better results in terms of line production rate as compared to static priority rules, such as always prioritizing the first or the second machine.
- When the transfer line is not balanced, e.g., the first machine substantially outperforms the second machine in terms of isolated production rate due to either a higher processing rate, a higher repair rate or a lower failure rate (see Eq. (5)), the optimal repair rule is to give priority to the second machine in order to maximize the line production rate.

We further showed that the additional line production rate gained by adding a second technician becomes smaller as the production line becomes less balanced. Thus, it is not always beneficial to add an extra technician.

The current work can be extended to production lines with multiple machines and technicians. In cases in which the number of machines that are down is greater than the number of technicians, a repair priority rule needs to be applied. The integrative solution approach suggested by Kuhn [23] can be extended in order to establish a preemptive repair discipline that depends on the number of items in the buffer.

Another direction for future research is to investigate cases with machines that have multiple failure modes, e.g., an electrical machine with two modes of failure, open circuit or short circuit failure. Levantesi et al. [27] proposed a two-machine line model with multiple failure modes; extending this model to consider a resource constraint (limited number of technicians) is challenging. In the case of multiple failure modes, the repair priority rule will be determined by a two-dimensional array, since a different priority rule needs to be determined for each combination of failure types (failure of the first machine and failure of the second machine).

Acknowledgments

The authors would like to thank Dr. Hassin R., Dr. Henig M. and the anonymous reviewers for their valuable comments and suggestions.

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