Partial Vertical Ownership, Capacity Investment and Information Exchange in a Supply Chain

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Abstract

Partial vertical ownership describes a situation in which a firm holds financial shares in either its supplier (referred to as partial backward integration) or its customer (partial forward integration). We study the effect of such financial interconnectedness on two operational decisions: capacity investment and information exchange. In our model, a retailer, who has superior information about the future market demand, has passive financial holdings in the supplier. Although this passive financial investment does not enable the retailer to directly influence the supplier's operational decisions, it does affect the market equilibrium. Specifically, financial interconnectedness between the firms can result in the retailer financing the entire capacity in the market. In addition, we characterize the conditions that ensure that information between the retailer and the supplier can be exchanged via *cheap-talk* communication. Interestingly, high level of information asymmetry facilitates the exchange of information via cheap-talk in the presence of these financial links. When cheap talk is not possible, we study the separating equilibrium that is achieved through the retailer's commitment to order in advance. In this case, the separating quantity can either increase or decrease with the level partial vertical ownership, and this trend does not depend the actual level of the financial holdings. We further analyze the incentive of the retailer to conceal demand information by choosing a pooling equilibrium, and conclude with discussing the effect of the financial interconnectedness on the parties' operational payoffs.

1 Introduction

Research in the area of supply-chain management has mainly focused on analyzing two structures of supply chains: centralized and decentralized. In the former case, one decision maker determines all decisions across the entire supply chain, whereas in the latter case, each firm in the supply chain is managed by an independent decision maker. Usually, the goal of each decision maker in the decentralized case is to maximize its own payoff, even at the expense of ignoring the consequences of these decisions on the performance of the entire supply chain. Scholars working in the area of operations management (OM) have devoted considerable effort to comparing the market outcome for these two supply-chain structures across key decisions, as well as to suggesting ways of remedying the possible efficiency loss that can result from the decentralized supply-chain structure (e.g., Cachon and Lariviere 2001, Cachon 2003).

In addition to these two possible structures of vertical supply chains, an intermediate level of interconnectedness between the firms in a supply chain may exist, referred to as *partial vertical ownership* (PVO). This interconnectedness describes a situation in which a firm holds financial shares in either its supplier (referred to as *partial backward integration*) or its customer (*partial forward integration*). Empirical research and anecdotal evidence show that such supply-chain structures are common in many industries (e.g., Reiffen 1998, Allen and Phillips 2000, Fee et al. 2006, Gilo and Spiegel 2011, Chen et al. 2017, Levy et al. 2018, Fang et al. 2021).[1] While PVO affords the holding firm a claim on the target firm's profit, in many cases it does not provide the holding firm with control rights, implying that the latter cannot directly influence the target firm's decisions (Greenlee and Raskovich 2006, Law Right 2016); in the present study, we focus on such a case.

Being aware of the existence of sophisticated financial structures in the market, scholars have also empirically examined whether firms strategically adapt their decisions in the presence of financial interconnectedness. Recent empirical evidence suggests that managers do take financial structures into consideration; Azar (2012), Semov (2017), He and Huang (2017), Schmalz (2018) and Lu et al. (2021) are a few recent examples that document the effect of financial structures on issues such as product design, market growth, competition level and advertising expenditure.

The aforementioned avenues of research lay the foundations of this study: an intermediate level of interconnectedness can exist between firms in a supply chain (via financial links), and this interconnectedness can affect the decisions of managers of firms that operate in such a supply chain. In this paper, we set out to examine how PVO affects two key operational decisions in a newsvendor supply chain: the ability to exchange information under information asymmetry and capacity investment. In our setting,

[1] For example, Toyota Motors, the core assembly firm of the Toyota production group, owns 25% of Denso, a large auto parts supplier. See Denso's State of Holders https://www.denso.com/global/en/investors/stock/overview/ (last accessed March 17, 2020).

we examine a PVO that provides no control over the target's decisions. This assumption is commonly adopted in the literature and is also observed empirically (e.g., Greenlee and Raskovich 2006, Chen et al. 2017, Levy et al. 2018, Fang et al. 2021). Nonetheless, this paper demonstrates that PVO can have an important indirect impact on the decisions of firms in a supply chain.

To study these issues, we develop a model (reminiscent of the prior work of Cachon and Lariviere 2001, and Özer and Wei 2006) in which a retailer ("she") has superior forecast information about the future stochastic demand than the supplier ("he"). The retailer can convey this information to the supplier in order to affect the capacity that the latter secures. We extend the existing literature by focusing on the case of a retailer who holds financial shares in her supplier (i.e., we examine the case of partial backward integration).

We demonstrate that under this asymmetric-information setting, meaningful information can, in some cases, be shared via cheap-talk communication. Cheap-talk information exchange refers to a situation in which private information is exchanged without any direct effect on the payoff of the retailer or the supplier (Crawford and Sobel 1982); nonetheless, in a *meaningful* cheap-talk equilibrium, firms do take the exchanged information into consideration when they determine their operational decisions. Prior research has demonstrated that in situations similar to ours, but absent financial links, meaningful cheap-talk information exchange cannot take place (see, e.g., Özer et al. 2011). Our work demonstrates that in the presence of financial connections, cheap-talk information exchange can actually emerge as an equilibrium. In our setting, when the forecast demand is high the retailer always shares information truthfully. However, when the forecast demand is low and the retailer contemplates whether or not to mislead the supplier into building a high capacity, two competing effects are at play. First, the retailer is aware that her operational profit would increase with the level of capacity that the supplier builds, which encourages her to mislead the supplier into building a high capacity. However, such an action would reduce the supplier's profit due to building capacity not according to the actual demand and, therefore, would disadvantage the retailer through the financial-holdings channel. In some cases, the latter effect outweighs the former, and cheap-talk information exchange can emerge as an equilibrium.

When cheap-talk information exchange does not exist, we study information exchange using the signaling tool of advance purchase. In this signaling game, the retailer commits to purchasing a certain quantity in advance; this quantity signals the state of demand to the supplier in a credible manner. We illustrate how financial holdings influence the signaling game. Here, two main effects are observed as well. First, when the retailer has a financial investment in the supplier, any financial loss on the part of the supplier influences the retailer's accounting value via the financial holdings channel. Thus, PVO reduces the retailer's incentives for information manipulation and better aligns the incentives of the retailer with those of the supplier. However, the second effect is that the "effective cost" of committing in advance to purchasing one unit decreases with the level of financial holdings because the retailer receives partial compensation for each purchased unit via the financial-holdings channel. Consequently, the second effect results in the need of the retailer to commit to purchasing a high quantity in advance in order to be perceived as accountable. As a result, there are cases in which the separating quantity increases with the level of PVO while in other cases this quantity decreases. We further compare the outcome of the signaling equilibrium with that of a pooling equilibrium. In the pooling equilibrium, the retailer acts in such a way that no information is revealed to the supplier. Using the undefeated equilibrium refinement (Mailath et al. 1993), we characterize the conditions that determine which of these two equilibria will be played.

Our model sheds light on the practical relevance of considering partial vertical ownership in a supply chain context with information exchange, and its main finding is also aligned with prior empirical studies. Dyer and Ouchi (1993) and Aziz and Tolouei (2012) list stock ownership in a supplier as one of the trust-building practices that facilitates exchange of information. Kwon and Suh (2004) also studied factors that lead to trust among supply-chain partners, and they write that a firm that owns a partner's stock takes into consideration the financial success of the business partner, as is also reflected in our model. In an interview to Harvard Business Review (Magretta 1998), Michael Dell said that knowledge sharing with partners is the foundation of Dell Inc.'s efforts toward "virtual integration"; Dell President and COO Kevin B. Rollins further said that such knowledge sharing includes information regarding planning and forecasting, and Dyer and Hatch (2004) write that one of the steps taken by Dell to achieve this goal is holding minority stakes in a few key vendors. In an extensive empirical research, Fee et al. (2006) examine more than 10,000 vertical relationships and determine the factors that result in the decision of a downstream firm to acquire stakes in its supplier. As an important factor to this decision they list the issue of asymmetric information and state that equity stakes may serve a role in encouraging information sharing. Finally, Drees et al. (2013) conclude, based on their empirical study, that corporate block ownerships can align incentives and mitigate information problems in corporate business relationships.

To summarize, we view the contribution of this paper along the following dimensions. First, we introduce an operational view of PVO through investigation of its effect on decisions such as information sharing and capacity investment. Although the interface between finance and operations has attracted the attention of OM scholars in recent years, this particular perspective has received relatively little attention. Specifically, we present two important findings regarding information sharing in supply chains. First, we present a new setting (PVO) in which firms are able to share information via cheap talk and we analyze the properties of this equilibrium. Second, when cheap-talk information exchange is not possible, we characterize the signaling game that results in credible information exchange. We find that in some circumstances, a pooling equilibrium (i.e., the concealment of information) is preferable to a separating equilibrium. We further show that PVO can result in the decision of the retailer to finance the entire capacity in the market, which improves the supply-chain's efficiency relative to a setting with no financial links.

2 Related Literature

This work is related to two main avenues of research. The first studies the ability to exchange information in a supply chain, as well as the value of, and the incentives for, such an undertaking. The second examines the interface between finance and operations management. In this section, we briefly describe the relationship between the contributions of our study and the current state of knowledge in the aforementioned areas of research.

Information sharing in a supply chain has been advocated by OM scholars as a means of matching supply with demand, especially in industries characterized by high levels of uncertainty. Extensive reviews of the value of information sharing can be found in Chen (2003) and more recently in Ha and Tang (2017). While the general consensus is that information sharing increases the overall efficiency in a centralized supply chain, a number of obstacles exist in reaching this outcome in a decentralized supply chain. For example, a firm may find the concealment of information to be beneficial when it anticipates that sharing information would result in a disadvantageous equilibrium (Li 2002, Li and Zhang 2002, Li and Zhang 2008). Furthermore, when information is unverifiable, firms can be tempted to distort the information that they communicate in order to increase their own profit at the expense of other firms in the supply chain. Lee et al. (2000), Cohen et al. (2003) and Özer and Wei (2006) provide anecdotal evidence of such information manipulation. To motivate firms in a supply chain to exchange information truthfully, researchers have suggested using sophisticated mechanisms such as screening contracts and signaling (e.g., Cachon and Lariviere 2001, Özer and Wei 2006). In this work, we follow this line of research and assume that information can be distorted if it benefits the more informed party (i.e., the retailer). Therefore, to align the incentives of the parties such that they exchange information in a credible manner, we analyze a signaling mechanism (when cheap-talk equilibrium is not supported) and investigate how financial links affect this signaling game.

Interestingly, in spite of the suggestions to use these sophisticated mechanisms to exchange information, empirical research has demonstrated that in many instances, firms, even under settings of information asymmetry, are able to exchange information using cheap talk (e.g., Desai and Srinivasan 1995, Bajari and Tadelis 2001, Iyer and Villas-Boas 2003). In an attempt to reconcile this apparent inconsistency, scholars have focused on characterizing situations that can arise in credible information exchange via cheap talk. For example: Ren et al. (2010) show that cheap talk can be supported when the parties contract over an infinite horizon and a review policy is implemented; Özer et al. (2011) demonstrate how trust and trustworthiness can assist in information exchange; Shamir and Shin (2016) illustrate that when two competing supply chains operate in the market, making information publicly available can result in cheap-talk information exchange; Chu et al. (2017) highlight the fact that when multiple decisions are being determined based on the shared information, cheap talk can be achieved; and Berman et al. (2019) study the role of operational flexibility in facilitating truthful information exchange via cheap talk. Our work contributes to this body of research by suggesting a new setting that supports the credible exchange of information via cheap talk – when financial links exist between the firms in a supply chain.

As mentioned, the second main stream of research related to our study is the interface between finance and operations management. OM scholars have studied the effect of financial topics such as debt (Chod and Zhou 2014, Chod 2017, Iancu et al. 2017, Alan and Gaur 2018), initial public offering (Babich and Sobel 2004), cash and time constraints (Yoo et al. 2016a, Yoo et al. 2016b, Bhaskaran et al. 2020), and trade credit (Peura et al. 2017, Tunca and Zhu 2018, Yang and Birge 2018) on operational decisions such as capacity choice and product development. Compared with these issues, the effect of financial structures on operational decisions has received relatively little attention by OM scholars with the following few exceptions. Aviv and Shamir (2021) study the impact of financial structure on the incentives for information acquisition and sharing. However, their paper differs from ours along two important dimensions: the market structure and the financial links. While we study a dyadic channel, Aviv and Shamir (2021) consider a supply chain with two competing retailers sourcing from a single supplier. In addition, they assume that the financial links are between the competing retailers (i.e., horizontal financial links), while we study financial links between a retailer and its supplier (i.e., vertical financial links). In common with our work, Fang et al. (2021) investigate vertical financial links, but similar to Aviv and Shamir (2021), they model a market with competition at the retail level. In particular, they study the effect of PVO on the competition level and they do not consider asymmetric information, which is the central theme of our work.

3 Model Formulation

3.1 The General Market

We study a supply chain consisting of a single retailer who sources a product component from a supplier. Due to the lengthy process of building capacity, the supplier must decide on the capacity level, K, prior to the selling season. We assume that the marginal cost of securing capacity is constant, and we denote it by c. We further assume that the retailer can observe the capacity investment of the supplier, and that the cost c is also common knowledge. Upon observing the demand realization, D, the retailer places an order with the supplier, and the latter produces it at a constant marginal cost (which is normalized to 0) in order to satisfy the demand, up to his capacity level.

The payment from the retailer to the supplier depends on the contract terms. In the main model, we assume that a simple wholesale-price contract is used, according to which the retailer pays a fixed sum, w, for each unit provided by the supplier.[2] Upon receiving the order from the supplier, the retailer sells the

[2] The assumption of an exogenous wholesale price is relaxed in Appendix B, where we assume that the supplier can offer a menu of contracts to the retailer.

product in the consumer market for a fixed retail price, *r*, where $r > w > c$ to ensure that the product is profitable for both parties. We further assume that the unmet demand is lost without any additional cost, and that unsold inventory has zero salvage value.

3.2 Information

Prior to the selling season, the retailer and the supplier share the prior forecast that the demand can be drawn from two possible distributions, reflecting high (index *H*) and low (index *L*) market conditions respectively. The distributions differ only in their mean, so the demand for a given market condition is captured by

$$D_i = \mu_i + \varepsilon,\ i \in \{L, H\}, \tag{1}$$

where $0 < \mu_L < \mu_H$, and ε is a zero-mean continuous random variable with a cumulative distribution function $F(x)$, where $\bar{F}(x) = 1 - F(x)$, and a probability density function $f(x)$. In particular, ε is defined over the support $(\underline{\varepsilon}, \bar{\varepsilon})$, where $\underline{\varepsilon} < 0 < \bar{\varepsilon}$, and it is further assumed that $\mu_L + \underline{\varepsilon} \geq 0$, to ensure non-negative demand values. The supplier and the retailer ascribe the probability of *p* to the high-market condition and the complement probability $(1-p)$ to the low-market condition.

In addition to the common knowledge of the retailer and the supplier, due to the proximity of the retailer to the market, we assume that she can observe the market condition (that is, whether the mean demand is μ_H or μ_L) prior to the decision of the supplier on the capacity investment. We further assume that the fact that the retailer can obtain this information is common knowledge. Many other papers that study asymmetric information adopt similar assumptions about the superior forecasting capabilities of the retailer (e.g., Cachon and Lariviere 2001, Li 2002, Özer and Wei 2006, Li and Zhang 2008, Shamir and Shin 2018). We further assume that this forecast information is non-verifiable and non-contractible.

Let Δ denote the difference between the two possible mean states of the demand (i.e., $\Delta \equiv \mu_H - \mu_L$). Since the retailer observes the precise value of the market state while the supplier does not, the value of Δ can be viewed as a measure of the level of *information asymmetry*. In a Cournot-competition setting, Anand and Goyal (2009), Kong et al. (2013), and Aviv and Shamir (2021) used the ratio between the two possible states of the demand as a measure of the level of information asymmetry.

The sequence of events in our model, as presented in Figure 1, is as follows: 1) The prior demand forecast is available to both parties. 2) The retailer observes the actual market condition. 3) The retailer shares information with the supplier, but this information may differ from the observed information if the retailer deems that distorting the information would be in her best interest. In addition, the retailer may place an advance order, denoted by q^{adv}, with the supplier. This advance order can be used to signal the condition of the market to the supplier. 4) The supplier, based on the shared information and the advance order (if it exists), establishes a posterior belief about the market condition and secures a capacity level K [3] at a cost cK. 5) The retailer observes the demand realization and orders a quantity $q = \min(K, \max(q^{adv}, D))$. 6) The supplier fills the order and receives a payment of wq. 7) The retailer sells the ordered quantity in the market.

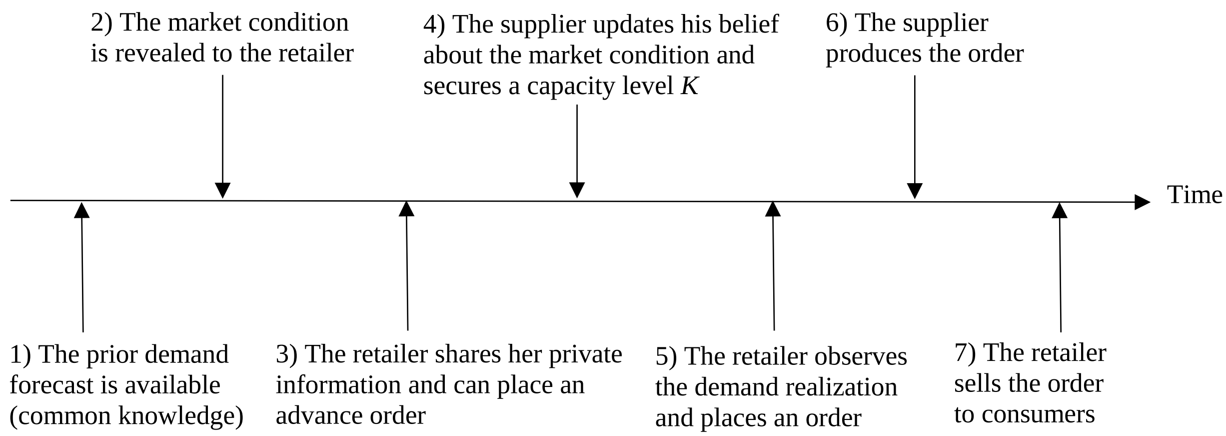


Figure 1. Sequence of events.

We adopt two important assumptions regarding the wholesale price. First, the wholesale price in our model is fixed. The use of an exogenous wholesale price is common in models with an endogenous capacity level and asymmetric information (see, for example, Anand and Goyal 2009, Özer et al. 2011, and Shamir and Shin 2016). In other papers, which instead adopt the assumption of an endogenously determined wholesale price, the capacity is usually determined exogenously (e.g., Ha and Tong 2008). Nevertheless, in Appendix B (see electronic companion), we relax the assumption of an exogenous wholesale price, and we study a mechanism in which the supplier determines the transfer prices in a strategic manner. The second assumption is that the price the retailer pays for units ordered in advance is identical to the price of units that are purchased after the capacity has been determined. This assumption is adopted to simplify the model analysis, and it does not qualitatively affect our main results.

3.3 Operational Payoffs

The retailer, if she chooses to, orders in advance a quantity q^{adv}. As described above, the supplier secures a capacity level of $K(\geq q^{adv})$ at a cost cK and receives a revenue of w for each unit sold to the retailer. Thus, the expected profit of the supplier for a given market condition μ_i and advance-order quantity q^{adv}

[3] Because $w > c$, the supplier will secure a capacity that satisfies $K \geq q^{adv}$.

is denoted by

$$\pi_s(K \mid \mu_i, q^{adv}) \equiv E\Big[w\min(K, \max(\mu_i + \varepsilon, q^{adv})) - cK\Big]$$
$$= w\left(K - \int_{q^{adv}-\mu_i}^{K-\mu_i} F(x)dx\right) - cK = w\left(z_s K - \int_{q^{adv}-\mu_i}^{K-\mu_i} F(x)dx\right),\ \ i \in \{L, H\}, \tag{2}$$

where $z_s \equiv 1 - c/w$ is the supplier's *critical fractile* (i.e., the ratio between the underage cost and the sum of the underage and overage costs). Upon demand realization, the retailer sells in the market the quantity $\min(K, \mu_i + \varepsilon)$ and receives for each unit the profit margin $r - w.$ If the demand realization is lower than q^{adv}, the retailer needs to pay the supplier for the excess units that she has committed to purchasing, an amount given by $\max(q^{adv} - \mu_i - \varepsilon, 0)$. We further denote the ex-ante payoff of the supplier by $\Pi_s = E[\pi_s],$ where the expectation is taken with respect to the two possible market conditions.

The expected operational profit of the retailer for a given μ_i is

$$\pi_r(q^{adv}, K \mid \mu_i) \equiv E\Big[(r - w)\min(\mu_i + \varepsilon, K) - w\max(q^{adv} - \mu_i - \varepsilon, 0)\Big]$$
$$= (r - w)\left(K - \int_{\underline{\varepsilon}}^{K-\mu_i} F(x)dx\right) - w\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i} F(x)dx. \tag{3}$$

We denote the ex-ante operational payoff of the retailer by $\Pi_r = E[\pi_r]$, where the expectation is taken with respect to the two possible market conditions.

The expected profit of the supply chain for a given μ_i is

$$\pi_{sc}(K \mid \mu_i) \equiv \pi_s(K \mid \mu_i, q^{adv}) + \pi_r(q^{adv}, K \mid \mu_i) = r\left(z_{sc}K - \int_{\underline{\varepsilon}}^{K-\mu_i} F(x)dx\right)$$

where $z_{sc} \equiv 1 - c/r$. Note that the expected profit of the supply chain is independent of the wholesale price w and the advance order q^{adv}.

3.4 Partial Vertical Ownership and Accounting Payoff

Because the key purpose of our research is to study the influence of existing partial vertical-ownership links on the way in which the market equilibrium is established and information is disseminated, we consider a setting in which the retailer has already purchased shares of the supplier. In particular, the purchase is conducted (exogenously) before the selling season and prior to observing any information about the demand.[4] Furthermore, in this paper, we focus on the effect of passive financial links, thus, we deny the

[4] In Appendix C, we relax this assumption and study the incentives to purchase and sell these shares.

retailer the ability to influence directly the decisions of the supplier by means of acquisition of control rights. This assumption has also been adopted in previous literature and has been observed empirically (e.g., Greenlee and Raskovich 2006, Levy et al. 2018, Fang et al. 2021). Consequently, we assume that these shares do not allow the retailer to dictate to the supplier his operational decisions (i.e., to influence his capacity level). However, since the retailer's profitability is financially linked to the supplier's profitability through the holding of shares, the operational actions of the two parties are intricately linked.

The effect of partial vertical ownership comes into play in our model through the retailer’s accounting payoff function:

$$\begin{aligned} v_r(q^{adv}, K \mid \mu_i) &= \pi_r(q^{adv}, K \mid \mu_i) + \beta\pi_s(K \mid \mu_i, q^{adv}) \\ &= r\left(\mu_i + \underline{\varepsilon} + \int_{\underline{\varepsilon}}^{K-\mu_i} \overline{F}(x)dx\right) - w(1-\beta)\left(q^{adv} + \int_{q^{adv}-\mu_i}^{K-\mu_i} \overline{F}(x)dx\right) - \beta cK, \end{aligned} \tag{4}$$

where $\beta \in [0,1]$ serves as a proxy measure for the level of partial vertical ownership (i.e., proportion of shares held). The explicit mathematical derivation of Equations (2)-(4), as well as all proofs, are relegated to Appendix E in the electronic companion.

As can be seen, the accounting payoff function weighs between the operating profits of the two firms in the supply chain. This approach for describing the behavior of firms that have financial links is common in the economics and OM literature. For examples in the economics literature, see Gilo et al. (2006), Shelegia and Spiegel (2012), and Levy et al. (2018). In the OM literature, Lai et al. (2011), Lai et al. (2012), and Lai and Xiao (2018) use this approach to capture the operational-decision behavior of a manager with a financial investment in the company he manages. Aviv and Shamir (2021) use a similar formulation to study markets with horizontal links between competing retailers.

The second line in Equation (4) shows that the accounting payoff of the retailer is comprised of three elements: the first describes the revenue that the retailer generates by selling to consumers; the second describes the *effective purchasing cost*; and the third reflects the reduction in the retailer’s accounting payoff due to the supplier’s capacity building cost. Note that due to the financial interconnectedness, the retailer’s effective purchasing cost per unit, $w(1-\beta)$, decreases in β because the retailer is compensated for each purchased unit to an extent that depends on the level of PVO. In a manner similar to the operational payoff, we use the notation $V_r = E\left[v_r\right]$ to denote the ex-ante accounting payoff of the retailer.

4 Capacity Financing under Complete Information

To better understand the effect of asymmetric information on firms' decisions in a supply chain, we start by providing a benchmark in which both the supplier and the retailer observe the market condition. Under complete information (denoted by the superscript *CI*), the supplier will secure a capacity level

$$K_i^{CI}(q^{adv}) = \begin{cases} \mu_i + F^{-1}(z_s) & q^{adv} \le \mu_i + F^{-1}(z_s) \\ q^{adv} & \text{otherwise} \end{cases}, \; i \in \{L, H\}. \tag{5}$$

The result in equation (5) can be interpreted as follows: The optimal capacity level from the supplier's perspective, absent any advance order, is $\mu_i + F^{-1}(z_s)$. Any level of advance order that is below this quantity will not alter the supplier's capacity decision. However, if the retailer orders above this level, the supplier simply matches the capacity level to the advance order of the retailer.

The retailer understands that ordering in advance a positive quantity up to $\mu_i + F^{-1}(z_s)$ will not change the capacity the supplier builds. Thus, under complete information, the decision to not place an advance order dominates the decision to order any positive quantity up to $\mu_i + F^{-1}(z_s)$. The retailer may consider ordering a quantity greater than $\mu_i + F^{-1}(z_s)$ in advance, if she finds the supplier's optimal capacity (absent any advance order) to be limiting. Let q^* be the optimal advance order made by the retailer when she finances the entire capacity in the market such that:

$$q^* = \arg\max_{q^{adv}} \left\{ v_r(q^{adv}, q^{adv} \mid \mu_i) \right\}. \tag{6}$$

Therefore, the retailer's optimal advance order under complete information is

$$q^{CI} = \begin{cases} 0 & \text{if } v_r(0, \mu_i + F^{-1}(z_s) \mid \mu_i) > v_r(q^*, q^* \mid \mu_i) \\ q^* & \text{otherwise} \end{cases}. \tag{7}$$

Thus, the retailer needs to consider between the following two options: $v_r(0, \mu_i + F^{-1}(z_s) \mid \mu_i)$ and $v_r(q^*, q^* \mid \mu_i)$. If the first payoff would be greater than the second, the retailer is satisfied with the capacity that the supplier builds, implying that the advance order quantity is zero. Otherwise, the retailer commits to purchasing a quantity q^* in advance to ensure the supplier will build a higher capacity level than that which he would have built absent the advance order. The following result explicitly characterizes the equilibrium outcome under complete information as a function of the level of partial vertical ownership, β, and shows that when β is sufficiently high, the retailer finances the entire capacity.

Theorem 1. **The retailer's incentive to finance the entire capacity under complete information**

Under complete information, the equilibrium advance order and capacity level are

$$\left(q_i^{CI}, K_i^{CI}\right) = \begin{cases} \left(0\ ,\ \mu_i + F^{-1}(z_s)\right) & \beta < \underline{\beta} \\ \left(\mu_i + F^{-1}(z_r(\beta))\ ,\ \mu_i + F^{-1}(z_r(\beta))\right) & \beta \geq \underline{\beta} \end{cases}, \tag{8}$$

where

$$\underline{\beta} \equiv \min_{\beta\in[0,1]} \left\{ \beta \;\middle|\; \begin{array}{l} z_r(\beta)\left(F^{-1}(z_r(\beta)) - F^{-1}(z_s)\right) \\ +\left(z_r + \beta(1-z_r)\right) \displaystyle\int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx \geq 0 \end{array} \right\} < 1 \tag{9}$$

with

$$z_r(\beta) \equiv 1-(1-z_r)(1-\beta z_s) = z_r + \beta z_s(1-z_r), \tag{10}$$

and

$$z_r \equiv z_r(0) = 1 - w/r \tag{11}$$

such that z_r *reflects the retailer's critical fractile without financial holdings.*

Although the optimal capacity level, from the supplier's perspective, is independent of the level of financial ownership (β), this level has an important effect on the retailer's decision as to whether to finance the entire capacity in the market or be satisfied with the supplier's capacity level. As β increases, the optimal capacity level from the retailer's perspective (q^*) also increases, so the retailer finds the supplier's capacity level to be more and more restrictive. This result is explained as follows: as β increases, the retailer's cost of financing one unit of capacity decreases, because she receives a higher compensation (through the financial-holdings channel) for each purchased unit. In particular, the effective cost of purchasing one unit of capacity is $w-\beta(w-c)$, which comprises the retailer's operational cost of purchasing one unit of capacity (w) as well as the benefit she receives from the supplier's success through the financial-holdings channel $(\beta(w-c))$. The critical threshold above which it is better off for the retailer to finance the entire capacity level in the market, $\underline{\beta}$, is always lower than 1. Thus, for any set of economic parameters, some level of financial holdings (possibly zero) ensures that the retailer will finance the entire capacity in the market.

This result is also illustrated in Figure 2. For low values of β $(\beta < 0.389)$, the optimal capacity of the supplier exceeds that of the retailer, so the retailer does not order any units in advance. For intermediate values of β $(0.389 \leq \beta < 0.855)$, the retailer would have built a higher capacity than that offered by the supplier based on her economic parameters. However, the retailer prefers the limited free capacity of the supplier over incurring the cost of financing the entire capacity. Therefore, in this region, the retailer also orders zero units in advance. Finally, when the level of β is very high $(\beta \geq 0.855)$, the retailer views the supplier's capacity as too limited, and she orders a strictly positive quantity in advance; thus, the retailer actually finances the entire capacity in the market.

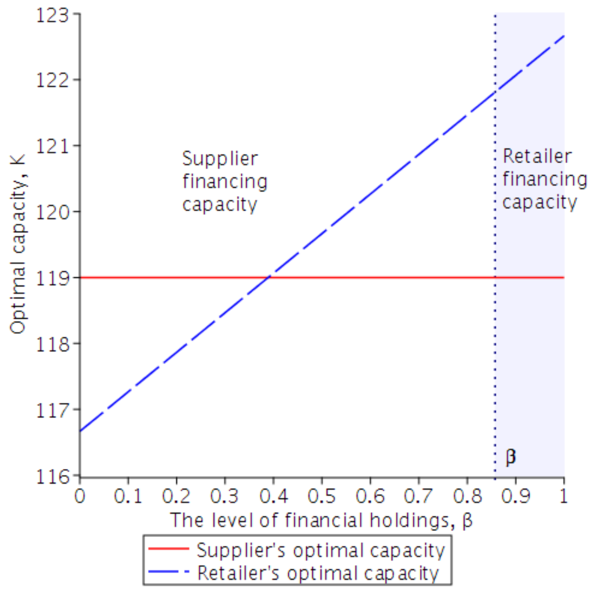


Figure 2. Optimal capacity for ε ~U(-10,10), c =11, w =20, r =30, μ_L =110 and μ_H =120.

In the following sections, we analyze the asymmetric situation. In this case, the retailer observes the market condition, and the supplier can establish a posterior belief about the market condition based on the actions of the retailer. These actions may consist of sharing information verbally (cheap-talk information exchange) or committing to purchasing a certain quantity in advance (signaling). We also study a pooling equilibrium, in which no information is revealed.

5 Can Meaningful Information be Exchanged via Cheap Talk?

Cheap-talk information sharing describes a situation in which the exchange of information does not directly affect the payoff of the retailer, nor that of the supplier (Crawford and Sobel 1982).[5] We are interested in exploring the ability of the parties in the supply chain to exchange *meaningful* information (i.e., share the true market condition) via cheap talk. To do so, the following conditions should be satisfied:

$$v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_H) \geq v_r(0, \mu_L + F^{-1}(z_s) \mid \mu_H) \tag{12}$$

$$v_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L) \geq v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L)\,. \tag{13}$$

[5] A cheap-talk equilibrium always exists when the retailer shares a message that is unrelated to her private information and the supplier simply ignores the shared information and builds capacity based on the prior belief (known as a babbling equilibrium).

Condition (12) ensures that when the retailer observes high market demand, she will truthfully share this information with the supplier. The left-hand side (LHS) denotes the retailer's accounting payoff when the supplier builds capacity according to the high market demand, whereas the right-hand side (RHS) denotes the retailer's accounting payoff when she misleads the supplier by stating that demand is low. Condition (13) ensures that the retailer also wishes to share information truthfully when she observes low market demand. In this condition, the LHS denotes the retailer's accounting payoff when she truthfully shares that demand is low, whereas the RHS denotes her accounting payoff when she misleads the supplier with the claim that demand is high.

In our model, condition (12) is always satisfied. When demand is high, misleading the supplier into believing that demand is low hurts the retailer in two ways. First, such a manipulation results in lower capacity, which reduces the retailer's operational profit (i.e., π_r). In addition, it hurts the supplier's profit because he does not build the optimal capacity level for the true market state. This reduction in the supplier's financial performance in turn hurts the retailer via the financial-holdings channel. Consequently, when demand is high, the retailer will always report the demand state truthfully. Thus, the ability to share meaningful information via cheap talk hinges upon satisfying condition (13). When the retailer has a financial interest in the supplier and the demand is low, two opposing effects influence the retailer's decision as to whether to exchange information truthfully. The first is that an increase in the capacity level secured by the supplier increases the retailer's operational profit (i.e., π_r) . This effect induces the retailer to mislead the supplier into believing that the demand is actually high, in the absence of partial vertical ownership (i.e., when $\beta = 0$). For the case we investigate, there is also an opposing effect. Misleading the supplier into believing that demand is high reduces the supplier's profits because he does not build capacity according to the real market demand; this misalignment, in turn, influences the retailer's payoff through her financial holdings.

The following result illustrates that, in some cases, meaningful information can be exchanged via cheap talk. It also illustrates the role of the financial holdings level, as well as the role of the information asymmetry level, in achieving this outcome.

Theorem 2. ***PVO and cheap-talk information exchange***

(i) Sharing the true market condition via cheap talk is possible when $\beta \geq \underline{\underline{\beta}}$ *, where*

$$\underline{\underline{\beta}} \equiv \frac{\pi_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L) - \pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L)}{\pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0) - \pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, 0)} = \frac{z_r}{1-z_r}\left(\frac{1-z_s}{R(\Delta)-z_s} - 1\right), \quad (14)$$

and

$$R(\Delta) \equiv \frac{1}{\Delta} \int_{F^{-1}(z_s)}^{\Delta + F^{-1}(z_s)} F(x)dx < 1, \quad (15)$$

(ii) Sharing the true market condition via cheap talk is possible only if $\Delta \geq \underline{\underline{\Delta}}$ *, where*

$$\underline{\underline{\Delta}} \equiv R^{-1}\left(z_s + \frac{1-z_s}{1+\beta(1/z_r - 1)} \right) \qquad (16)$$

and $R^{-1}(\cdot)$ *is the inverse function of* $R(\Delta)$ *.*

Theorem 2(i) illustrates that a high level of PVO results in the ability to exchange meaningful information via cheap talk — an outcome that cannot occur absent financial holdings. The threshold value $\underline{\underline{\beta}}$ reflects the ratio between the retailer's operational gain and the supplier's loss when he is misled into believing that demand is high when it is actually low. It also upholds that if the supplier's loss as a result of the manipulation is severe compared with the retailer's operational gain and if, in addition, the financial holdings of the retailer are sufficiently high, then the retailer has the incentive to share information truthfully with the supplier even when demand is low.

Theorem 2(ii) allows us to relate the gap between the two possible market conditions (Δ) to the ability to exchange information via cheap talk. The theorem states that when this gap is large, achieving meaningful information sharing in equilibrium is possible, whereas when this gap is small, no meaningful cheap-talk information sharing can be achieved. The relationship between the ability to share information via cheap talk and the gap between the two market states is, likewise, based on the two effects that were described above. When the gap between the two states is large, the operational benefit of the low-type retailer from misleading the supplier into building a high capacity level is high, because the retailer benefits from this higher capacity level. However, the marginal operational benefit from each additional unit of capacity decreases in the value of Δ, because the likelihood of using this additional unit of capacity to increase the sales volume is low when the gap in market states is large. At the same time, the financial loss incurred by the retailer due to misleading the supplier increases in the value of Δ. This is because, as the gap increases, the supplier builds a higher level of capacity, which is inappropriate for the low market demand. Consequently, the supplier suffers severe financial losses that affect the retailer's portfolio value. Thus, when Δ is sufficiently high, achieving cheap-talk information exchange is feasible.

Previous research has demonstrated that when a retailer and a supplier interact using a simple wholesale-price contract over one selling season, meaningful information-sharing cannot be achieved via cheap talk (Özer et al. 2011). In spite of these predictions, empirical evidence suggests that in many cases, firms do exchange information via cheap talk when using this contract (e.g., Desai and Srinivasan 1995, Bajari and Tadelis 2001, Iyer and Villas-Boas 2003, Cohen et al. 2003). In an attempt to explain these empirical findings, researchers have provided several justifications for the ability to share information via cheap talk (Ren et al. 2010, Özer et al. 2011, Shamir and Shin 2016, Chu et al. 2017, Berman et al. 2019), as described in Section 2. Theorem 2 serves as an important addition to this line of research. It shows that even when the relationship between the parties lasts only one selling season, the parties use a simple wholesale-price contract, and there is no assumption of trust between the parties, information can be shared via cheap talk. Such an outcome can be achieved when the retailer has a financial interest in the supplier.

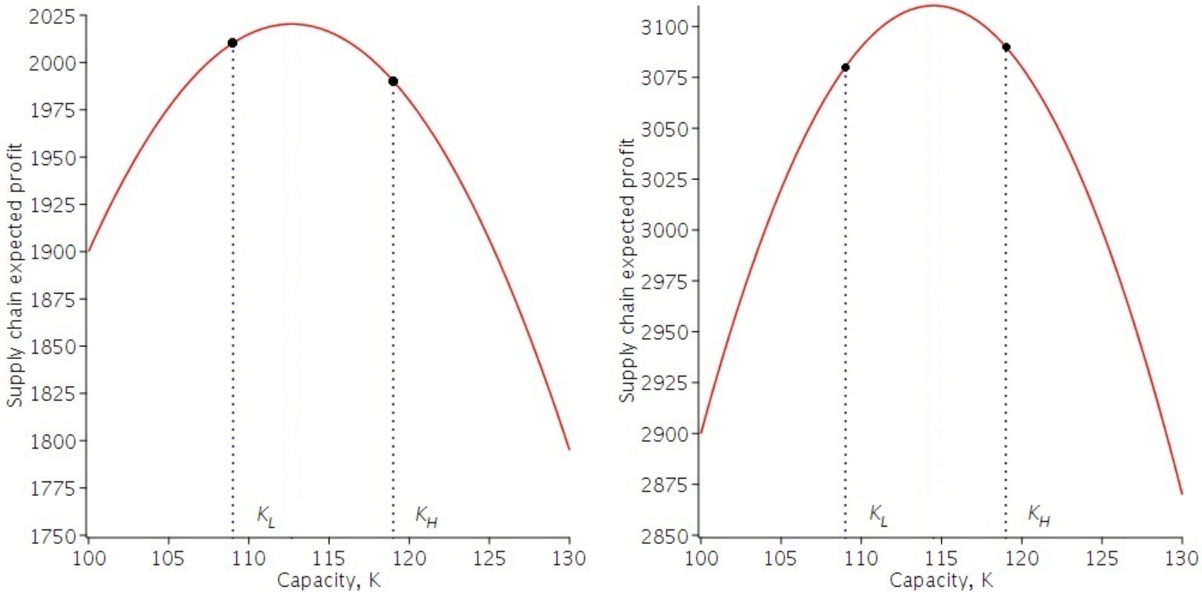
We note that the threshold value $\underline{\underline{\beta}}$ that determines the ability to share meaningful information via cheap talk can be higher than 1. This would reflect a situation in which, regardless of the level of financial holdings, cheap talk cannot be used to exchange meaningful information (since $\beta \leq 1$). In such a case, when the market state is low, the retailer's operational gain from pretending to be a high type is always greater than the supplier's corresponding loss. Therefore, the question naturally arises as to whether it is possible to find a condition that ensures the feasibility of a cheap-talk equilibrium. The next proposition relates the operational performance of the supply chain to the ability to exchange information via cheap talk.

Proposition 1. ***Supply-chain performance and cheap talk***

Meaningful information can be exchanged via cheap talk (i.e., $\underline{\underline{\beta}} < 1$) if and only if $\pi_{sc}(\mu_H + F^{-1}(z_s) \mid \mu_L) < \pi_{sc}(\mu_L + F^{-1}(z_s) \mid \mu_L)$.

The condition outlined in Proposition 1 relates the performance of the entire supply chain during the low market state to the ability to exchange information via cheap talk. The proposition upholds that if, in the low market state, the supply chain is better off with the supplier's high capacity level (i.e., $K_H = \mu_H + F^{-1}(z_s)$) than with his low capacity level (i.e., $K_L = \mu_L + F^{-1}(z_s)$), then cheap-talk information exchange cannot be achieved for any level of financial holdings, while if the reverse holds, then it is possible to achieve cheap-talk information exchange for any $\underline{\underline{\beta}} \leq \beta \leq 1$. Figure 3 illustrates the results stated in Proposition 1. In order to determine whether cheap talk can be achieved (for some value of $\beta < 1$), we compare, for the low-market state, the operational performance of the supply chain under the supplier's low capacity and his high capacity. In Panel (a), the supply chain is better off with the low capacity of the supplier than with the capacity that the supplier builds upon believing that the demand is actually high. This case represents a scenario in which cheap talk is feasible. In contrast, in Panel (b), the supply chain is better off with the high capacity of the supplier even though the market state is low. Therefore, in this case, for any value of $\beta < 1$, the low-type retailer will prefer to mislead the supplier, and consequently, cheap talk cannot be achieved.

Further results regarding the effect of the information gap on the ability to exchange information via cheap-talk is provided in Appendix D (see electronic companion).



(a) (b)

Figure 3. Supply chain expected profit for ε ~U(-10,10), c=11, w=20, μ_L =110 and μ_H =120.
Panel (a) r=30 ($\underline{\underline{\beta}}<1$); Panel (b) r=40 ($\underline{\underline{\beta}}>1$)

6 Advance Order as a Signaling Mechanism

What happens when cheap talk cannot be supported as an equilibrium? In this section, we follow the previous literature that has studied mechanisms of exchanging information in a credible way. Specifically, we concentrate on the mechanism of an advance purchase, according to which the retailer orders a certain quantity in advance to signal the state of the demand to the supplier. This kind of interaction leads to a signaling game (Gibbons 1992), and we focus on characterizing a separating equilibrium using pure strategies. In this case, upon observing the state of the demand, each type of retailer orders a different quantity, so the supplier can perfectly infer the demand state based on the retailer's order. The analysis of the advance purchase as a signaling tool is relevant when $\beta < \min\{\underline{\beta}, \underline{\underline{\beta}}\}$ because otherwise, either cheap talk is supported in equilibrium or the retailer prefers to finance the entire capacity in the market.

When $\beta < \min\{\underline{\beta}, \underline{\underline{\beta}}\}$, under the symmetric-information case, the low-type retailer prefers to order zero units in advance, and relies upon the capacity that the supplier builds. Therefore, in the retailer's efficient separating equilibrium (which satisfies the intuitive criterion introduced by Cho and Kreps 1987), when revealing her identity, the low-type retailer will order zero units in advance. In this equilibrium, the high-type retailer needs to order in advance a quantity that ensures that the low-type retailer would not wish to mimic her. The following theorem provides a full characterization of this separating equilibrium.

Theorem 3. ***The separating equilibrium***

Under the separating equilibrium:

(i) The retailer orders in advance $q^{adv}(\mu_i) = \begin{cases} 0 & \text{if } \mu_i = \mu_L; \\ \tilde{q} & \text{if } \mu_i = \mu_H, \end{cases}$

where $\tilde{q}$ *(* $\tilde{q} < \mu_H + F^{-1}(z_s)$ *) satisfies*

$$\int_{\varepsilon}^{\tilde{q}-\mu_L} F(x)dx = \frac{\Delta\left[z_r(1-z_s) - \left(z_r + \beta(1-z_r)\right)\left(R(\Delta) - z_s\right)\right]}{(1-\beta)(1-z_r)} = \frac{\Delta(1-z_s)}{1+\underline{\underline{\beta}}(1/z_r - 1)} \cdot \left(1 - \frac{1-\underline{\underline{\beta}}}{1-\beta}\right). \quad (17)$$

(ii) The supplier builds the capacity of

$$K(q^{adv}) = \begin{cases} \mu_L + F^{-1}(z_s) & \text{if } q^{adv} < \tilde{q}; \\ \mu_H + F^{-1}(z_s) & \text{if } q^{adv} \in [\tilde{q}, \mu_H + F^{-1}(z_s)]; \\ q^{adv} & \text{if } q^{adv} > \mu_H + F^{-1}(z_s), \end{cases}$$

supported by the belief system that $P(\mu_i = \mu_H) = \begin{cases} 0 & \text{if } q^{adv} < \tilde{q}; \\ 1 & \text{if } q^{adv} \geq \tilde{q}. \end{cases}$

Under the separating equilibrium, characterized in Theorem 3, the low-type retailer chooses not to order any units in advance, whereas the high-type retailer orders a quantity $\tilde{q}$. Upon observing the advance-order quantity, the supplier establishes a posterior belief about the market state according to the following rule: The supplier interprets any advance order below $\tilde{q}$ as signaling a low market state, and any order quantity greater than or equal to $\tilde{q}$ as representing a high market state. Based on the new posterior belief, the supplier secures the appropriate capacity level. Since $\tilde{q} < \mu_H + F^{-1}(z_s)$, on the equilibrium path, the secured capacity in the market when demand is high is $\mu_H + F^{-1}(z_s)$.

Theorem 3 also illustrates the cost, from the retailer's perspective, of asymmetric information when signaling is required. In this case, the supplier sets the same capacity level as under complete information. However, when demand is high, the retailer needs, under asymmetric information, to commit to purchasing a quantity $\tilde{q}$ in order to secure the same capacity level as in the complete-information setting (understanding that there is some probability that not all $\tilde{q}$ units will be sold). Note that although the signaling game is costly for the retailer, it improves the supplier's profit. This is because, under the signaling game, the supplier builds the same capacity level as in the complete-information case, but receives a commitment for a minimum purchase quantity from the high-type retailer.

We next evaluate the effect of the level of financial holdings (β) on the separating quantity $\tilde{q}$.

Proposition 2. ***Financial holdings and the monotonicity of the separating quantity***

The separating quantity $\tilde{q}$ *increases (decreases) in the level of financial holdings* β *when* $\underline{\underline{\beta}} > 1$ *(* $\underline{\underline{\beta}} < 1$ *), and it is constant when* $\underline{\underline{\beta}} = 1$.

Interestingly, Proposition 2 states that the separating quantity either increases or decreases for the entire parameter region of β in which the signaling game is played. In a signaling game, when the low-type retailer has the incentive to mimic the high type, *upward distortion* is observed. Upward distortion is also observed in our model because the high-type retailer needs to commit to ordering in advance, whereas in the complete-information setting, she does not order any units when $\beta < \underline{\beta}$. Proposition 2 demonstrates that while the degree of upward distortion depends on the level of financial holdings, the trend itself, of whether this quantity is increasing or decreasing, is independent of the level of financial holdings. To understand this result in an intuitive way, note that β introduces two main effects on the separating quantity. We term the first effect "*incentive alignment*" and the second "*signaling effectiveness*". According to the first effect, the retailer's financial interest in the operational success of the supplier results in better alignment of the incentives of the two parties. More specifically, the retailer has a weaker incentive to mislead the supplier when β is higher, because when the supplier suffers operational loss (due to building a high capacity level when the demand state is actually low), the retailer is also affected by this loss via the financial-holdings channel. As for the second effect, credible information exchange stems from the commitment of the high-type retailer to purchasing a certain quantity in advance — a commitment that can result in a significant loss to the low-type retailer if she attempts to mislead the supplier. This loss occurs because (some of) the advance-order units might not be purchased under the low-market state, resulting in financial loss to the low-type retailer attempting to mislead the supplier. The effective cost, from the retailer's perspective, of each unsold unit decreases in β, as shown in Equation (4). Consequently, the effectiveness of each expected unsold unit as a signaling tool also decreases in β. Therefore, the second effect suggests that in order to achieve separation, the high-type retailer needs to increase her commitment level as β increases.

Figure 4 illustrates the two possible cases. In the first case ($r = 40$, implying that $\underline{\underline{\beta}} = 1.2 > 1$), the separating quantity increases in the level of financial holdings for all values of β, as illustrated by the dashed blue line. However, beyond the threshold level $\beta = \underline{\beta}$ (0.634 in the example), the retailer prefers to finance the entire capacity in the market than to order in advance the separating quantity (the continuum of the separating quantity is shown by the blue dots). In the second case ($r = 30$, implying that $\underline{\underline{\beta}} = 0.6 < 1$), the separating quantity decreases in β, as illustrated by the solid red line. At $\beta = 0.6$, the separating quantity equals zero, suggesting that from this threshold level up to $\beta = \underline{\beta}$ (0.855 in the example), the information is shared via cheap talk; this is illustrated by the continuum of the solid red line. Beyond that point, the retailer finances the entire capacity in the market (the continuum of the separating quantity is shown by the red dots).

Interestingly, the condition that determines whether the separating quantity increases or decreases in the level of financial holdings is related to the feasibility of cheap-talk information exchange (see Theorem 2). When cheap talk is feasible (i.e., $\underline{\underline{\beta}} < 1$), the separating quantity decreases in β, whereas when cheap talk cannot be supported (i.e., $\underline{\underline{\beta}} > 1$), the separating quantity increases. The following proposition sheds additional light on the condition under which the separating quantity decreases in β. It shows that this condition is related to both the separating quantity absent any financial holdings and the payoff of the supplier.

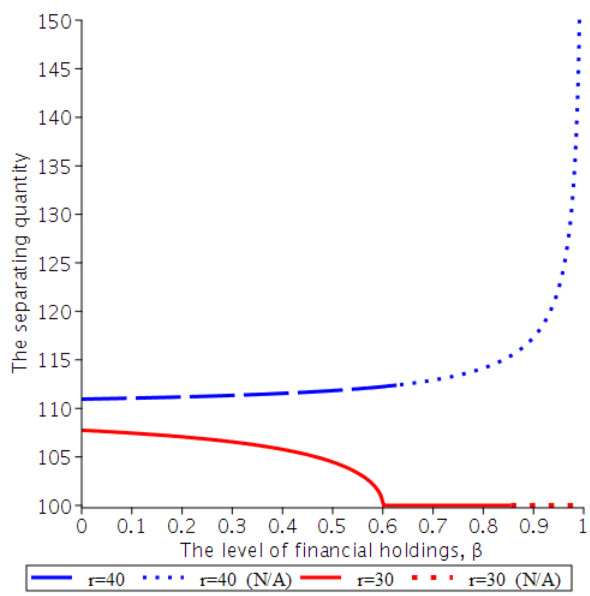


Figure 4. The separating quantity $\tilde{q}$ as a function of β for ε~U(-10,10), c=11, w=20, μ_L=110 and μ_H=120.

Proposition 3. ***Supplier's payoff and the monotonicity of the separating quantity***

The separating quantity $\tilde{q}$ decreases in β if and only if $\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, \tilde{q}_0) < \pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0)$, where $\tilde{q}_0$ is the separating quantity when $\beta = 0$ (i.e., $\int_{\underline{\varepsilon}}^{\tilde{q}_0 - \mu_L} F(x)dx = \Delta(1 - R(\Delta)) / (1/z_r - 1)$).

The quantity $\tilde{q}_0$ is the amount ordered by the high-type retailer when $\beta = 0$. Absent financial holdings, this advance order ensures that the low-type retailer is indifferent between ordering zero units in advance (and revealing her type), and committing to purchasing $\tilde{q}_0$ in advance and thereby misleading the supplier into believing that demand is high. The proposition relates the monotonicity direction of the separating quantity to the effect of information manipulation on the supplier absent any financial holdings. It states that if the supplier would be better off (worse off) being misled by the low-type retailer ordering the separating quantity $\tilde{q}_0$ when there are no financial holdings, then $\tilde{q}$ increases (decreases) in β.

In signaling games, the separating quantity is determined by the condition under which the sender is incentivized to reveal information truthfully. In our case, since the retailer also takes into consideration the payoff of the supplier, an interesting situation arises: we need to evaluate the effect of information manipulation on the receiver (i.e., supplier). If the receiver prefers information manipulation for the case of $\beta = 0,$ then when the level of financial holdings is strictly positive, the high-type retailer needs to signal the state of the demand using a higher advance order than in the case of no financial holdings. If the opposite holds, the high-type retailer is able to signal the state of the demand with a lower advance order than in the case of $\beta = 0$.

7 The Pooling Equilibrium

Section 6 discussed the ability of the retailer and the supplier to exchange information using the signaling mechanism. However, this mechanism can be costly to the high-type retailer since she needs to commit to purchasing a certain quantity in advance – a quantity that may not be sold in the market. In this section, we explore the alternative option of the retailer — to conceal information, meaning that both types of retailer would order the same quantity in advance. Such a strategy prevents the supplier from inferring the state of the demand. In the following proposition, we characterize such a pooling equilibrium.

Theorem 4. The pooling equilibrium

(i) There exists $\beta^P \in [0, \underline{\beta})$, such that for any $\beta < \beta^P$ a pooling equilibrium exists and it is characterized by the following properties:

(a) Both types of retailer order zero units in advance (i.e., $q^P = 0$).

(b) The supplier builds a capacity K^P, which satisfies:

$$pF(K^P - \mu_H) + (1-p)F(K^P - \mu_L) = z_s, \tag{18}$$

based on the following belief system: $P(\mu_i = \mu_H \mid q^P) = \begin{cases} p & \text{if } q^P < \overline{q}^P; \\ 1 & \text{if } q^P \geq \overline{q}^P, \end{cases}$

where $\overline{q}^P$ *satisfies:*

$$\int_{\underline{\varepsilon}}^{\overline{q}^P-\mu_H} F(x)dx = \frac{1}{w(1-\beta)}\left[(r-w(1-\beta z_s))(\mu_H + F^{-1}(z_s) - K^p) - (r-w(1-\beta))\int_{K^p-\mu_H}^{F^{-1}(z_s)} F(x)dx\right] \quad (19)$$

and β^P *satisfies:*

$$\beta^P \equiv \min_{\beta\in[0,1]}\left\{\beta \;\middle|\; \begin{array}{l} z_r(\beta)\left(F^{-1}(z_r(\beta)) - (K^P-\mu_H)\right) \\ +\left(z_r+\beta(1-z_r)\right)\displaystyle\int_{\underline{\varepsilon}}^{K^P-\mu_H} F(x)dx - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx \geq 0 \end{array}\right\} < \underline{\beta}. \quad (20)$$

(ii) If $\beta \geq \beta^P$ *then a pooling equilibrium does not exist.*

The proposition states that when the level of financial holdings is sufficiently high ($\beta \geq \beta^P$), a pooling equilibrium does not exist. In this case, the high-type retailer prefers to increase the capacity in the market by committing to purchasing in advance a high quantity — an outcome that the low-type retailer does not wish to pool with. When $\beta < \beta^P$, both retailer types order the same quantity in advance under the pooling equilibrium (zero units) and the supplier builds a capacity K^P based on the prior information, since no additional information is revealed through the advance order of zero units. According to the belief system outlined in the proposition, any order below the threshold $\overline{q}^P$ results in the supplier building a capacity based on the prior belief, and any order above this threshold (which is not observed on the equilibrium-path) convinces the supplier that demand is high.

When demand is high, the supplier secures a lower capacity level under the pooling equilibrium than under the separating equilibrium – an outcome that hurts the high-type retailer. On the other hand, in the pooling equilibrium, the high-type retailer does not place an advance order, and thus does not incur any of the signaling costs that are observed in the separating equilibrium. It is important to note that while the pooling equilibrium is a *perfect Bayesian equilibrium* (PBE), it does not satisfy the *intuitive criterion* (Cho and Kreps 1987), as is also the case for other games involving asymmetric information and only two possible types of sender (for some recent examples in an OM context, see Schmidt et al. 2015, Jiang et al. 2016, and Tian and Jiang 2017). In the next section, we discuss the equilibrium choice when both the separating and the pooling equilibrium are feasible.

8 Equilibrium Selection

The existence of several types of equilibria raises the question of which equilibrium will be chosen. As mentioned above, the intuitive criterion rules out the pooling equilibrium in our setting, although this equilibrium has some appealing properties, as will be shown below. Therefore, as an equilibrium refinement, we adopt a refinement known as the *undefeated equilibrium*, introduced by Mailath et al. (1993). According to this equilibrium refinement, the chosen equilibrium is the PBE that the high-type retailer finds most profitable because, in our setting, the low-type retailer attempts to mimic the high type, while it is the high-type retailer that attempts to signal the true demand state to the supplier. Furthermore, the low-type retailer always earns a higher payoff in the pooling equilibrium than in the separating equilibrium. Therefore, when both the separating and the pooling PBE exist, if the high-type retailer prefers the pooling equilibrium, then this is the chosen outcome according to the undefeated equilibrium refinement. A few other recent papers in OM have also adopted this equilibrium refinement so as to avoid eliminating the pooling equilibrium (Schmidt et al. 2015, Jiang et al. 2016, Tian and Jiang 2017, Aviv and Shamir 2021). The following proposition summarizes the outcome in our setting based on this equilibrium refinement.

Proposition 4. ***The effect of PVO on the equilibrium selection***

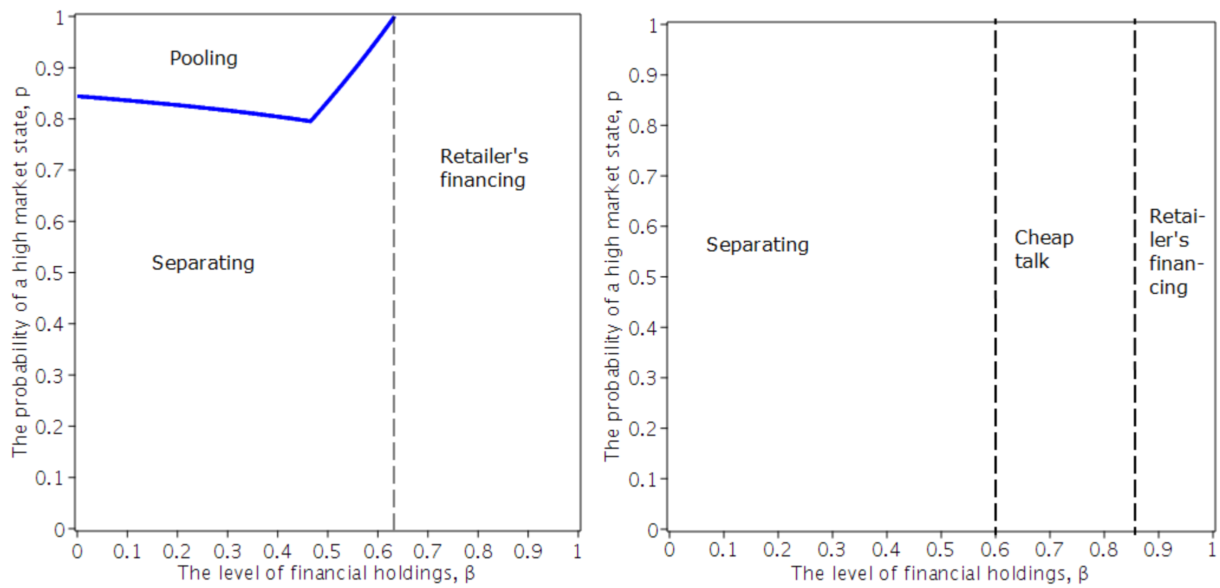
(i) *When* $\beta \geq \underline{\beta}$ *, the retailer finances the entire capacity in the market according to Theorem 1.*

(ii) *When* $\underline{\underline{\beta}} \leq \beta < \underline{\beta}$ *, cheap-talk information is exchanged according to Theorem 2(i).*

(iii) *When* $\beta < \min(\underline{\underline{\beta}}, \underline{\beta})$ *:*

- *(a)* *If* $\beta \geq \beta^P$ *then the separating equilibrium is played according to Theorem 3.*
- *(b)* *If* $\beta < \beta^P$ *then a value* $\bar{p}$ *exists such that for any* $p \leq \bar{p}$ *, the separating equilibrium is played according to Theorem 3, and for any* $p > \bar{p}$ *, the pooling equilibrium is played according to Theorem 4.*

When $\beta \geq \underline{\beta}$, regardless of the mechanism for information exchange, the retailer is not satisfied with the capacity that the supplier secures, and thus she chooses to finance the entire capacity in the market. When $\underline{\underline{\beta}} \leq \beta < \underline{\beta}$, the retailer is able to exchange information via cheap talk, and in this case, she prefers the supplier to finance the capacity in the market. When $\beta < \min(\underline{\underline{\beta}}, \underline{\beta})$, the retailer needs to choose between exchanging information via the signaling mechanism and concealing information via the pooling equilibrium. When both of these options are viable, the proposition highlights the role of the prior probability in determining the chosen outcome. When the signaling cost is high, and there is a high prior probability that demand is high, the high-type retailer prefers the pooling equilibrium, since, in this case, the supplier builds a relatively high capacity level without the need for the retailer to commit to purchasing costly units in advance. However, when the prior probability of high demand is low, under the pooling equilibrium, the supplier builds a limited capacity; thus, the high-type retailer prefers to incur the signaling cost in order to induce the supplier to secure a higher capacity level.



(a) (b)

Figure 5. The equilibrium as a function of β and p for ε~U(-10,10), c =11, w =20, μ_L =110 and μ_H =120 with (a) r =40 (and thus $\underline{\underline{\beta}} > 1$); (b) r =30 (and thus $\underline{\underline{\beta}} < 1$)

Figure 5 illustrates the results presented in Proposition 4. We depict the chosen equilibrium as a function of the level of financial holdings (horizontal axis) and the prior belief that demand is high (vertical axis). Panel (a) represents the scenario of $\underline{\underline{\beta}} > 1$. It shows that when the prior belief is high and the level of financial holdings is low, the pooling equilibrium will be played. When the prior belief is low, the separating equilibrium is played at low levels of β, but once the level of financial holdings reaches a critical value, the retailer prefers to finance the entire capacity in the market. Since $\underline{\underline{\beta}} > 1$ in this example, the separating quantity increases in β (see Proposition 2) and cheap talk is not feasible.

Panel (b) represents the scenario of $\underline{\underline{\beta}} < 1$. Recall that in this case, the separating quantity decreases in β and cheap talk is feasible. Further, the separating equilibrium, the cheap-talk equilibrium and the equilibrium in which the retailer finances the entire capacity, are all independent of the prior p. For the example depicted in Panel (b), the cost of signaling is so low that the separating equilibrium is always preferable over the pooling equilibrium. Consequently, in this case, for low values of the financial holdings, the separating equilibrium is played; for intermediate values, the cheap-talk equilibrium is played; and finally, for high values of the financial holdings, the retailer finances the entire capacity.

We next consider the effect of the wholesale price on the equilibrium outcome.

Proposition 5. ***The effect of the wholesale price on the equilibrium selection***

There exist w_1 *and* w_2, *where* $c < w_1 \leq w_2 < r$, *such that*

- *(i)* *If* $w \leq w_1$ *then the retailer finances the entire capacity in the market according to Theorem 1.*
- *(ii)* *If* $w_1 < w < w_2$ *then either the separating or the pooling equilibrium is played (see Theorems 2 and 3).*
- *(iii)* *If* $w \geq w_2$ *then cheap-talk information is exchanged according to Theorem 2(i).*

Proposition 5 illustrates the role of the wholesale price in determining the equilibrium outcome. When the wholesale price is low (specifically, when $w \leq w_1$), the capacity secured by the supplier is low, and consequently, the retailer prefers to finance the entire capacity. On the other hand, when the wholesale price is high ($w \geq w_2$), the supplier secures a high capacity level. In addition, in this region, since the profit margin of the retailer is low, the operational benefit from misleading the supplier to build capacity that does not match the actual demand is relatively small. Consequently, in this case, information can be exchanged via cheap talk. Finally, between these two thresholds, the retailer does not wish to finance the entire capacity and she is also unable to convey information using cheap talk. Therefore, in this region ($w_1 < w < w_2$), the retailer either signals information by committing to purchasing units in advance, as illustrated in Theorem 2, or she conceals information by playing the pooling equilibrium (and ordering zero units in advance). These results are depicted in Figure 6, which outlines the played equilibrium as a function of the wholesale price (the horizontal axis) and the level of PVO (the vertical axis).

Proposition 5 demonstrates that due to the effect of the wholesale price on the equilibrium outcome, the supplier's payoff is not continuous with respect to the wholesale price. For example, for low values of the wholesale price, the supplier incurs no risk, since the retailer finances the entire capacity in the market. Moving from this region ($w \leq w_1$) to the region $w_1 < w < w_2$ may actually hurt the supplier because it results in the supplier financing the capacity, while the retailer either commits to purchasing some units in advance (in the case of the separating equilibrium) or does not purchase any units in advance (in the case of the pooling equilibrium). Therefore, a small increase in the wholesale price can result in non-continuous reduction in the supplier's profit.

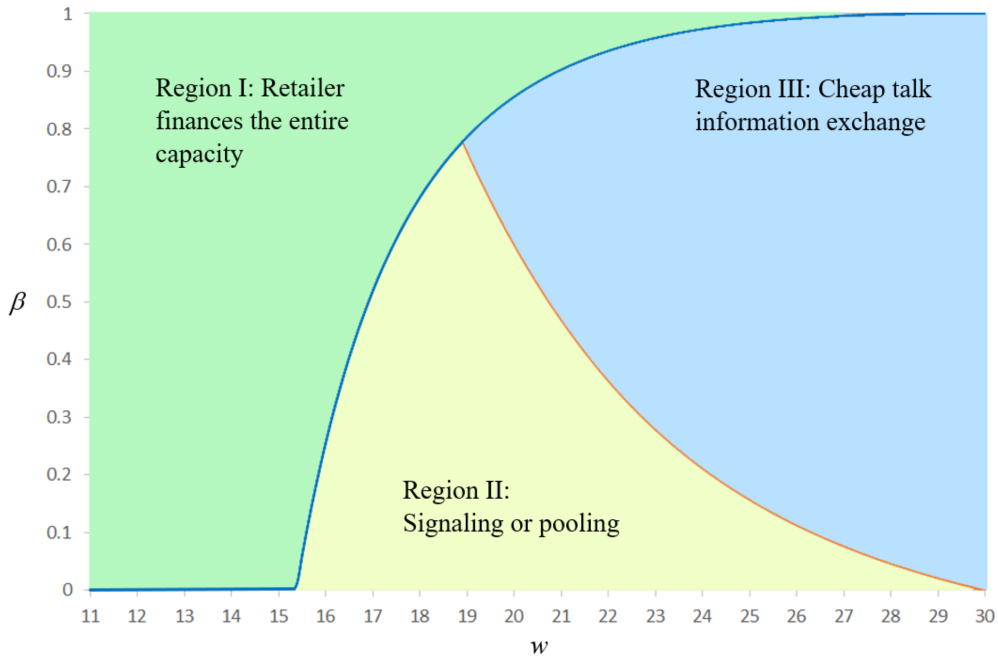


Figure 6. Equilibrium outcome as a function of the wholesale price and PVO level for ε~U(-10,10), c=11, r=30, μ_L=110 and μ_H=120

9 The Effect of Partial Vertical Ownership on the Firms' Payoffs

In this section, we study the effect of PVO on the payoffs of the firms in the supply chain. The analysis is performed from an ex-ante perspective, that is, taking into account the two possible demand states (according to the prior belief) and the equilibrium outcome. Holding a larger share in the supplier clearly benefits the retailer. Yet, a higher level of financial holdings is also associated with an increased cost of acquiring these shares – a cost that is not considered in the main model. Furthermore, our model does not include the decision of the retailer regarding whether or not to acquire shares in the supplier in the first place, because such a decision can be influenced by many other factors apart from the ability to share information within the supply chain. In Appendix C (see electronic companion), we extend our model and discuss the incentives of the retailer to (a) purchase shares at the beginning of the process and (b) sell her shares during the process (specifically, after observing the market demand). For the present purposes, to avoid evaluating the effect of holding a larger share in the supplier without modeling the cost of achieving it, we concentrate on examining the effect of β on the ex-ante *operational* payoff of the retailer. In addition, we study the effect of PVO on the supplier's ex-ante payoff. Further discussion regarding the effect of PVO on the consumers is provided in Appendix D.

Proposition 6. ***PVO and the firms' payoffs***

The effect of PVO on the retailer's and supplier's payoffs is as follows:

(i) *When* $\beta \geq \underline{\beta}$, Π_s *increases in* β *and* Π_r *decreases in* β.

(ii) *When* $\underline{\underline{\beta}} \leq \beta < \underline{\beta}$, *both* Π_s *and* Π_r *are independent of* β.

(iii) *When* $\beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\}$ *and the signaling game is played:*

(a) *If* $\underline{\underline{\beta}} \geq 1$, Π_s *increases and* Π_r *decreases in* β.

(b) *If* $\underline{\underline{\beta}} < 1$, Π_s *decreases and* Π_r *increases in* β.

(iv) *When* $\beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\}$ *and the pooling game is played, both* Π_s *and* Π_r *are independent of* β.

The first part of the proposition describes the effect of PVO when the retailer pays for the entire capacity level. In this domain, the supplier's payoff is deterministic, and increasing the level of PVO results in an increase in the capacity level that the retailer finances, which, in turn, improves the situation for the supplier. However, the situation is different when considering the retailer's operational profit. The retailer determines the capacity level by considering not only her own operational payoff, but also that of the supplier (due to the financial channel). As a result, the capacity exceeds the optimal level from the retailer's *operational* perspective. Consequently, as the level of β increases, the operational payoff of the retailer decreases.

The second part of the proposition corresponds to the case in which information is shared via cheap talk. In this scenario, the supplier determines the capacity level according to his own economic parameters, and the retailer does not purchase any units in advance. Consequently, the supplier's and retailer's payoffs are independent of the PVO level.

The last two parts of the proposition (iii and iv) correspond, respectively, to the situation in which information is shared using the signaling tool of an advance purchase, and the situation in which no information is shared due to the fact that the pooling equilibrium is chosen. When the signaling game is played, Proposition 6 demonstrates that two cases exist. When $\underline{\underline{\beta}} \geq 1$, the retailer's advance order increases in the PVO level, β. In this case, the supplier benefits from the increased advance order, because there is a lower risk that he will be left with unused capacity. Consequently, the supplier's payoff increases in the level of β. The fact that the advance order increases in β implies that, as the level of financial holdings increases, the retailer needs to commit to ordering a higher quantity in advance in order for the supplier to build the same capacity level. Consequently, the operational payoff of the retailer decreases in β. On the other hand, when $\underline{\underline{\beta}} < 1$, the signaling quantity decreases in the PVO level, β, implying that the retailer is able to credibly signal the state of demand using a lower quantity of units purchased in advance.

Consequently, the supplier is worse off as β increases, while the retailer is better off. Finally, when the pooling equilibrium is played, the retailer orders zero units in advance, and the capacity is determined based on the prior belief of the supplier. In this case, the capacity level, and consequently the supplier's and retailer's payoffs, are independent of β.

10 Discussion and Managerial Implications

The analyses of the different types of equilibria presented in this paper reveal a number of important properties regarding the effect of PVO on both the ability to exchange information and the overall capacity in the market. In this section, we summarize and further discuss these findings.

Who finances the market capacity? In our model, it is the supplier who builds the capacity in the market. However, the retailer can influence this decision through her commitment to purchase units in advance. Such a commitment can create a situation in which the retailer is the party in the supply chain that effectively finances the entire capacity in the market (and the supplier simply builds capacity to match this advance order). Our model shows that when the level of PVO is sufficiently high (specifically, when $\beta \geq \underline{\beta}$), the retailer will not be satisfied with the capacity the supplier intends to build, and will prefer to commit to purchasing in advance a higher quantity. The supplier, in turn, will then build this higher capacity. Thus, the introduction of PVO may change the system from one in which the supplier decides upon the market capacity to one where the market capacity is determined by the retailer. Such a change is beneficial to the supplier, who no longer incurs the risk of building units that go unutilized, and consumers benefit from the increased capacity in the market as well.

In addition, we show that when the level of financial holdings is sufficiently high, the introduction of asymmetric information does not affect the equilibrium outcome. That is, in both the symmetric- and asymmetric-information cases, the equilibrium outcome is identical – the retailer commits in advance to purchasing a quantity that effectively finances the entire capacity in the market.

The ability to share information via cheap talk. An important conclusion of our model is that financial holdings provide the retailer and the supplier with the ability (under certain conditions) to exchange information by means of cheap talk. Normative models have shown that in settings similar to ours, but absent financial interconnectedness, meaningful cheap-talk information cannot be exchanged because the retailer has an interest in manipulating the supplier to secure a high capacity level when demand is low (Özer et al. 2011). Nevertheless, empirical evidence demonstrates that, in many instances, firms do exchange information verbally (for some recent examples, see Özer et al. 2018, Özer and Zheng 2019, and Li et al. 2020). The current study adds to the recent scholarly effort to provide explanations for this empirical evidence (other explanations are provided by Ren et al. 2010, Özer et al. 2011, Shamir and Shin 2016, Chu et al. 2017, and Berman et al. 2019) by showing that PVO may align the incentives of the retailer and the supplier, such that cheap-talk information exchange can be achieved. This result is consistent with prior research that has demonstrated empirically and theoretically that PVO can align the interests of the target and acquirer (Allen and Phillips 2000, Greenlee and Raskovich 2006) and can facilitate cooperation (Allen and Phillips 2000, Fee et al. 2006). In addition, Dyer and Ouchi (1993) argue that part of the success of the Japanese Auto manufacturing industry can be attributed to the special business partnerships formed between companies and their suppliers. One of the trust-building practices reported in their study is the ownership of stocks in the supplier. The authors argue that this mechanism implies that the firms have made a commitment to each other, and that, consequently, they have weaker incentives to take advantage of each other. The model we present is consistent with this empirical evidence and demonstrates that PVO can encourage cooperation and information exchange.

Our model predicts that cheap-talk equilibrium can be achieved when the level of information asymmetry is high. In this case, inducing the supplier to build a high capacity level, when the market state is low, hurts the retailer via her financial-holdings channel. This financial loss can outweigh the operational benefit of the high capacity, thus incentivizing the retailer to share the true demand state with the supplier via cheap talk. Usually, settings with a high level of information asymmetry represent a challenge for the credible exchange of information. Avinadav and Shamir (2021) study a similar setting to ours, but without PVO. They show that no cheap-talk equilibrium exists and that in order to exchange information in a credible manner, the retailer must commit to purchasing in advance a quantity that increases with the level of information asymmetry – an outcome that is costly for the retailer. Thus, the tool of financial holdings may facilitate the credible exchange of information and allow the retailer to share information without incurring high signaling costs.

PVO and signaling/pooling. When cheap-talk equilibrium cannot be achieved, this paper studies the credible exchange of information by using an advance order as the signaling tool. We show that while the separating quantity is monotone in the level of financial holdings, the direction of this monotonicity is independent of the PVO level, and it is determined by other economic factors in the market. Therefore, there are cases in which, to exchange information credibly, the separating quantity increases with the level of PVO – a situation that hurts the operational payoff of the retailer. In the alternative case, the separating quantity decreases with the level of PVO – a situation that improves the operational payoff of the retailer but hurts the supplier.

We further demonstrate that exchanging information is not always in the best interests of the retailer. When the signaling cost is high and the prior probability of a favorable market condition is also high, the retailer may prefer a pooling equilibrium over the separating equilibrium. In the pooling equilibrium, the supplier secures the capacity based on the prior belief, and the retailer does not need to incur the signaling cost of an advance order. We characterize the conditions under which the retailer prefers to conceal information than to signal the market condition.

Policy implications. For more than a century, the acquisition of partial ownership was examined by antitrust authorities. For example, The Clayton Act of 1914 states that:[6]

> "*no person engaged in commerce or in any activity affecting commerce shall acquire, directly or indirectly, the whole or any part of the stock or other share capital and no person subject to the jurisdiction of the Federal Trade Commission shall acquire the whole or any part of the assets of another person engaged also in commerce or in any activity affecting commerce, where [...] the effect of such acquisition maybe substantially to lessen competition, or to tend to create a monopoly*"

While most of the investigations of antitrust authorities were aimed at examining mergers between competing firms, in recent years the scope of these investigations was extended to also account for the cases of partial vertical ownership. In 2013, the European Commission recommended to[7]

> "*extend the scope of the Merger Regulation to give the Commission the option to intervene in a limited number of problematic cases of structural links, in particular those creating structural links between competitors or in a vertical relationship*",

Where the term structural links is used to describe the case of partial ownership. Our work can also contribute in the debate regarding the efficiency effects of such practice. Our model suggests that when the level of PVO is high (specifically, when $\beta \geq \underline{\beta}$) the retailer chooses to finance the entire capacity in the market. Such an action results in higher market capacity compared with the case of no PVO and it mitigates the problem of double marginalization.

Summary. Scholars working in the area of OM have devoted considerable effort to evaluating the performance of centralized and decentralized supply chains. In this work, we argue that in addition to these two important supply-chain structures, an intermediate level of interconnectedness may exist between firms in a supply chain. This interconnectedness is achieved when one firm has a financial interest in the other firm.

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[6] The Clayton Act is now part of the U.S code Section 15. See https://www.law.cornell.edu/uscode/text/15/18# (last accessed November 21, 2021).

[7] See https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52014DC0449&rid=2 (last accessed November 21, 2021).

Our work demonstrates the impact of such a financial structure on the ability of the firms in a supply chain to exchange information and on the capacity level that is secured in the market. In Particular, we demonstrate that there are cases in which the market capacity increases with the level of financial holdings – an outcome that improves the overall efficiency of the supply chain by alleviating the problem of double marginalization (Spengler 1950). In other cases, such financial interconnectedness results in the ability to exchange information via cheap talk, and when cheap talk is not feasible, it influences the advance order quantity placed by the retailer to achieve the credible exchange of information.

In addition to its contribution to the area of information exchange in supply chains, this paper adds a new dimension to the field of the interface between OM and finance – a topic that has received considerable attention in recent years. Our work adds to this fruitful research area through its demonstration of the critical effect of financial interconnectedness on operational decisions.

Appendix A. Glossary of Notations

Variable	Description
D	Demand (random variable)
ε	The random element of the demand for a given market state; it has zero mean and is defined over the support $(\underline{\varepsilon},\overline{\varepsilon})$

Decision variable	Description
K	Capacity level – set by the supplier
K^{CI}	Capacity level under complete information at equilibrium
K^{P}	Capacity level under a pooling equilibrium
q^{adv}	Advance order – set by the retailer
q^{*}	The optimal advance order placed by the retailer when she has to finance the entire capacity in the market
q^{CI}	The optimal advance order under complete information at equilibrium

Parameter	Description
β	Level of financial holdings
c	Marginal cost of securing capacity
w	Unit wholesale price
r	Unit retail price
μ_i	Expected demand for market state $i \in \{L,H\}$
Δ	Difference between the two possible mean states of the demand
z_s	Supplier's critical fractile
z_r	Retailer's critical fractile for $\beta = 0$
z_{sc}	Supply chain's critical fractile

Function	Description
F	Cumulative distribution function of ε
f	Probability density function of ε
$z_r(\beta)$	Retailer's critical fractile for a given β
$\underline{\beta}$	The critical threshold above which the retailer chooses to finance the entire capacity level in the market
$\underline{\underline{\beta}}$	The critical threshold above which cheap talk is possible

β^P	The critical threshold below which a pooling equilibrium exists
$\tilde{q}$	The separating quantity that ensures the low-type retailer does not wish to mimic the high-type retailer
$\tilde{q}_0$	The separating quantity when $\beta = 0$
$\overline{q}^P$	The advance-order quantity below which the supplier maintains his prior belief about demand

Profit function	Description
$\pi_s(K \mid \mu_i, q^{adv})$	Supplier's expected profit for a given market state and advance order
$\pi_r(q^{adv}, K \mid \mu_i)$	Retailer's expected operational profit for a given market state
$\pi_{sc}(K \mid \mu_i)$	Expected operational profit of the supply chain for a given market state
$v_r(q^{adv}, K \mid \mu_i)$	Retailer's expected accounting payoff function
Π_s	Ex-ante payoff of the supplier
Π_r	Ex-ante operational payoff of the retailer
V_r	Ex-ante accounting payoff of the retailer

Appendix B. Screening Contracts

In the main model, we assumed that the retailer signals the state of the demand by committing to purchasing a certain quantity in advance. In such a setting, it is the retailer who takes the initiative and designs the signaling mechanism such that information is exchanged at the lowest possible cost (to her). In addition, we assumed that the wholesale price is exogenous, regardless of the realized market state. In this appendix, we relax these two assumptions and explore the effect of PVO on the contracting scheme when it is the supplier that designs a set of screening contracts. In settings of asymmetric information, a screening contract means that the uninformed party offers a set of contracts to the informed party, and the choice of a specific contract by the informed party conveys information to the uninformed party (examples of such contracts in an OM setting can be found in Ha 2001, Özer and Wei 2006, and Avinadav et al. 2020, 2021).

Specifically, let us assume that the supplier offers a menu of two possible contracts.[8] Based on the revelation principle (Myerson 1979), we denote this menu by $\{(K_H, T_H), (K_L, T_L)\}$. The menu is designed to operate as follows: upon observing high demand, the retailer will choose the contract (K_H, T_H), while in the case of low demand, she will choose the contract (K_L, T_L). Once a specific contract (K_i, T_i), where $i \in \{L, H\}$, is chosen, the supplier builds capacity K_i and the retailer pays the sum T_i.[9]

To reflect this new setting, we make some changes to the notations of the payoff functions of the retailer and the supplier (compared with the main model). In the new contract, the operational payoff of the retailer is given by $\pi_r(K_i, T_i \mid \mu_j) = rE\left[\min(D, K_i) \mid \mu_j\right] - T_i$. When the retailer observes the market state $j \in \{L, H\}$, and chooses contract (K_i, T_i), she understands that the market capacity will be K_i and the total payment to the supplier will be T_i. The operational payoff of the supplier is given by $\pi_s(K_i, T_i) = T_i - cK_i$; note that it does not depend on the actual market realization.

To better understand the effect of PVO on the design of this menu of contracts, we start by characterizing the outcome when $\beta = 0$. We denote this outcome using the superscript 0.

Lemma B1. *When $\beta = 0$, the optimal menu of contracts designed by the supplier is determined as follows:*

(i) K_H^0 is the solution of the equation $\frac{d}{dK}\left(E\left[\min(D, K) \mid \mu_H\right]\right) = \frac{c}{r}$.

(ii) K_L^0 is the solution of the equation

$$\frac{d}{dK}\left(E\left[\min(D, K) \mid \mu_L\right]\right) - \frac{p}{1-p}\frac{d}{dK}\left(E\left[\min(D, K) \mid \mu_H\right] - E\left[\min(D, K) \mid \mu_L\right]\right) = \frac{c}{r}.$$

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[8] This is without loss of generality, since we assume that there are only two possible states of demand. We also assume that the parameter values are such that it is in the best interests of the supplier to operate under both states of demand.

[9] It is also possible to consider a linear wholesale price w_i, instead of a one-time fee of T_i, by substituting $T_i = w_i K_i$.

(iii) $T_L^0 = rE\left[\min(D, K_L^0) \mid \mu_L\right]$.

(iv) $T_H^0 = r\left\{E\left[\min(D, K_H^0) \mid \mu_H\right] - \left(E\left[\min(D, K_L^0) \mid \mu_H\right] - E\left[\min(D, K_L^0) \mid \mu_L\right]\right)\right\}$.

This well-known solution exhibits the following properties. First, the supplier offers, in the high-demand state, the capacity that maximizes the overall performance of the supply chain ("efficiency at the top"). In contrast, in the low-demand state, the supplier offers a lower capacity than that required to maximize the overall value of the supply chain ("distortion at the bottom"). In the low-demand state, the supplier extracts the entire surplus from the retailer, such that the payoff of the retailer is zero. In the high-demand state, the supplier leaves some surplus for the retailer, as a means of ensuring that the latter does not choose the contract designed for the low-type retailer. The payoff of the high-type retailer is called information-rent, since the high-type retailer is paid to reveal her hidden type.

We now analyze the optimal menu of contracts designed by the supplier when $\beta > 0$. In a similar manner to the case of no financial holdings, two constraints are binding. The supplier must extract the entire surplus from the retailer in the low-demand state, while the supplier must provide information-rent to the high-type retailer to deter her from choosing the alternative contract. Therefore, the supplier's problem is formulated in the following manner:

$$\begin{aligned}
&\max_{\{(K_H,T_H),(K_L,T_L)\}} \left\{ p(T_H - cK_H) + (1-p)(T_L - cK_L) \right\} \\
&S.t. \quad rE\left[\min(D, K_H) \mid \mu_H\right] - T_H + \beta(T_H - cK_H) = rE\left[\min(D, K_L) \mid \mu_H\right] - T_L + \beta(T_L - cK_L) \\
&\qquad\;\; rE\left[\min(D, K_L) \mid \mu_L\right] - T_L + \beta(T_L - cK_L) = 0
\end{aligned}$$

The first constraint ensures that the high-type retailer chooses the appropriate contract, while the second ensures that the low-type retailer will not decline any offered contract. The next proposition describes how PVO affects the properties of the contract designed by the supplier.

Proposition B1. *When* $\beta > 0$ *:*

(i) The capacity levels offered to the high-type retailer and the low-type retailer are the same as when $\beta = 0$.

(ii) The price charged by the supplier (in each market state) increases with β .

Interestingly, according to part (i), when β is positive, its value does not affect the capacity levels offered in the menu of contracts. However, as shown in part (ii), the *price* charged by the supplier, in each market state, increases with the level of financial holdings. In our model, the retailer receives additional compensation via the financial channel – a compensation that increases with β. Therefore, the supplier can ensure that the low-type retailer will not reject the contract even though she is charged a higher price than in the case of no financial holdings. Therefore, as the level of financial holdings increases, the supplier knows that he can charge the retailer a higher price while still ensuring that she accepts the contract. When demand is high, the supplier is also able to charge the retailer a higher price as β increases, but for a different reason. In this case, as the level of financial holdings increases, the retailer has a weaker incentive to attempt to mislead the supplier by choosing the contract designed for the low type. Consequently, the supplier is able to force the retailer to choose the contract that he has designated for her despite offering a lower information-rent (i.e., charging a higher price) compared to a setting with no financial holdings.

The above analysis reveals very different results compared to those of the main model. In the setting described in this appendix, the supplier is better off as the level of financial holdings increases, since he is able to charge a higher price for the same capacity level (in both demand states). In contrast, recall that in the main model, it was shown that if the level of financial holdings is positive, it can result in cheap-talk information exchange or a reduction in the quantity required to signal the state of the demand – outcomes that make the supplier worse off relative to the case of no financial holdings.

Appendix C. Retailer's Incentives to Purchase and Sell Shares

In the main model, we assumed that the level of shares held by the retailer is fixed and does not vary throughout the sequence of events. This assumption was adopted since, in many instances, decisions regarding the acquisition and sale of shares involve additional factors beyond the ability to exchange information in the supply chain – elements that are not modeled in this work. Note also that the approach used in our main model is consistent with much of the previous research that has studied the topic of PVO and has assumed an exogenous number of shares.

In this appendix, we relax the assumption of an exogenous level of financial holdings, and we study two issues: first, the retailer's incentive to sell shares in the supplier once she has obtained information about the market state (but prior to signaling using the advance-order tool); and second, the decision of the retailer to purchase shares in the first instance.

Selling Shares

Herein, we relax the assumption of exogenous financial holdings and assume that the retailer initially holds a share of β in the supplier, but is able to sell it (or part of it) after observing the state of the demand. The main conclusion of this section is that the act of selling (or refraining from selling) shares cannot signal any information to the market, nor to the supplier. Specifically, we show that both retailer types will choose to hold their shares prior to committing to purchasing units in advance.

We assume that the market determines the price of shares in the supplier based on all available information. This means that the market (i) understands that the retailer enjoys superior information about the state of the demand, and (ii) is able to make inferences regarding the demand state based on the actions of the retailer. In a potential separating equilibrium that is based on the action of selling (or refraining from selling) shares, this action provides a signal to the supplier/market, which immediately updates the value of these shares. We first show that such a separating equilibrium cannot exist.

Proposition C1. *There is no separating equilibrium in which one type of retailer sells shares and the other does not.*

The intuitive explanation of this result is as follows. In a separating equilibrium, one retailer type sells shares while the other does not. Two possible cases can emerge in such a separating equilibrium: either the low-type retailer sells some shares while the high type does not, or the converse occurs. The proposition states that neither of these cases can emerge as an equilibrium. First, assume that the low-type retailer, upon observing that demand is low, attempts to sell (some of) her shares, while the high type prefers not to sell. In this case, the market infers that the demand is low, and prices the shares accordingly. Furthermore, the supplier also infers that demand is low and sets the capacity level according to this market state. This scenario would not be beneficial to the low-type retailer, as she would achieve a higher payoff by not selling the shares. This is because the latter decision would lead the supplier to believe that demand is high and thereby set the appropriate capacity level for this market state – an outcome that benefits the low-type retailer. Consequently, a separating equilibrium in which the low type sells shares while the high type does not, cannot exist.

In the second possible separating equilibrium, the high-type retailer sells her shares, while the low-type retailer refrains from doing so. However, this equilibrium is also unfeasible, because the low-type retailer can (strictly) do better by mimicking the high type and selling her shares. By selling shares prior to demand realization, the low-type retailer gains two benefits: (i) she receives a higher payoff for her shares, compared to not selling them; and (ii) the supplier sets a higher capacity level, which increases the low-type retailer's operational payoff. The same arguments can be used to illustrate that any potential separating equilibrium in which the two types of retailer sell different numbers of shares cannot exist.

Since it is impossible for the action of selling shares to carry any informational value, the only possible equilibrium is a *pooling* equilibrium in which both retailer types sell the same number of shares. The following proposition illustrates that, in this equilibrium, both retailer types will hold all their shares.

Proposition C2. *In equilibrium, after observing the state of the demand, neither retailer type will sell any shares.*

After establishing that both retailer types will sell the same number of shares, Proposition C2 shows that this number will be zero. The intuitive explanation behind this result is as follows. Since both retailer types sell the same number of shares, the market price for these shares will be based on the prior belief regarding the state of the demand. In this case, the high type will prefer not to sell any shares because she knows that the demand state is high, and thus the value of the shares in the supplier is higher than the price she will receive for these shares. Consequently, no shares will be sold after observing the demand state. We next evaluate the incentive of the retailer to purchase shares ex-ante.

Ex-ante Incentive to Purchase Shares

Assume that prior to observing the state of the demand (high or low), the retailer is able to purchase some shares in the supplier. We assume that the retailer needs to pay the "fair" market value for these shares. This implies that to obtain a share of β in the supplier, the retailer will need to pay $\beta\,\Pi_s$, where Π_s is determined based on the market evaluation of the supplier, which takes into account all available ex-ante information. This assumption implies that in order to decide upon the optimal level of financial holdings, denoted by β^*, the retailer solves $\max_\beta \{V_r - \beta\,\Pi_s\}$.

Recall that the accounting payoff of the retailer, V_r, also includes the expected payoff of the supplier, such that $V_r - \beta\,\Pi_s = (\Pi_r + \beta\,\Pi_s) - \beta\,\Pi_s = \Pi_r$. This suggests that the retailer takes into consideration only the effect of β on her ex-ante *operational* payoff when she decides on the optimal level of PVO. Proposition 8 outlines the effect of the level of financial holdings on the operational payoff of the retailer. This proposition allows us to state the following:

Proposition C3. *When the retailer is able to decide on the ex-ante optimal level of financial holdings, she will choose the following:*

(i) When $\underline{\underline{\beta}} \geq 1$, *the retailer has no incentive to purchase any shares in the supplier, such that* $\beta^* = 0$.

(ii) When $\underline{\underline{\beta}} < 1$, *the retailer purchases* $\beta^* = \min\left\{\underline{\beta}, \underline{\underline{\beta}}\right\}$.

The proposition states that the number of shares purchased by the retailer depends on the value of the threshold $\underline{\underline{\beta}}$. Recall that when $\underline{\underline{\beta}} \geq 1$, the separating quantity increases with the level of financial holdings, which in turn, reduces the operational payoff of the retailer. Therefore, in this case, the retailer prefers to signal the state of the demand without any financial holdings (that is, $\beta^* = 0$).

In the alternative case, when $\underline{\underline{\beta}} < 1$, the separating quantity decreases with the level of financial holdings, which increases the operational payoff of the retailer. Therefore, in this case, the retailer will purchase the highest possible number of shares in order to maximize her operational payoff. The separating quantity reduces to zero at the threshold level of $\underline{\underline{\beta}}$ above which cheap talk is played (i.e., $\underline{\underline{\beta}} < \underline{\beta}$), and thus, in this case, the retailer will choose to purchase a share of $\underline{\underline{\beta}}$ in the supplier. If, however, $\underline{\underline{\beta}} \geq \underline{\beta}$ (meaning that cheap talk is not played), the retailer will choose to purchase a share of $\underline{\beta}$ in the supplier. Therefore, the retailer purchases shares at the level $\beta^* = \min\left\{\underline{\beta}, \underline{\underline{\beta}}\right\}$.

Proposition C3 implies that if the retailer decides strategically on the number of purchased shares, and if $\underline{\underline{\beta}} < 1$, then neither a pooling equilibrium nor a signaling equilibrium will be played on the equilibrium path. Rather, the observed outcome will be that, on the equilibrium path, either cheap-talk information is exchanged or the retailer finances the entire capacity in the market.

Appendix D. Additional Results

The effect of the information asymmetry level on the ability to exchange information

Here, we focus on the effect of the gap between the two possible market states (measured by Δ) on the separating quantity. To do so, we fix μ_L and we vary μ_H.

Proposition D1. ***Information-asymmetry level and the separating quantity***

The behavior of the separating quantity $\tilde{q}$ as a function of Δ (for a given μ_L) is as follows:

(i) If $\beta = 0$ then $\tilde{q}$ (weakly) increases in Δ.

(ii) If $\beta > 0$ then there exists $\underline{\Delta} \equiv F^{-1}\left(R\left(\underline{\Delta}\right)\right) - F^{-1}(z_s) > 0$, such that $\tilde{q}$ increases in Δ for $\Delta \in \left[0, \underline{\Delta}\right)$ and decreases in Δ for $\Delta > \underline{\Delta}$.

Proposition D1 describes the way in which the gap between the two possible market states influences the separating quantity. When Δ increases (while fixing μ_L), the gap between the capacity levels that the supplier builds for the two possible market states also increases. Avinadav and Shamir (2021) study the case of $\beta = 0$, and show that as the gap increases, to prevent the low-type retailer from mimicking the high type, the latter needs to commit in advance to ordering a higher quantity. Part (i) of the proposition claims that, for this case, the separating quantity (weakly) increases in Δ.

In contrast with the case of $\beta = 0$, when some level of financial holdings exists (i.e., $\beta > 0$), the separating quantity is unimodal. When the low-type retailer decides whether or not to mimic the high type, she considers two effects – operational and financial. An increase in the capacity level augments the operational profit of the retailer; however, such a manipulation also reduces the supplier's payoff, which hurts the retailer's accounting payoff via the financial-holdings channel. As the information gap between the two possible states increases, the latter effect increases while the former effect diminishes. As a result, when this gap is large, the low-type retailer will have a weak incentive to try to mislead the supplier. Consequently, the advance order required to induce a low-type retailer to truthfully reveal her type is low when the gap between the two possible demand states is sufficiently high.

Figure D1 illustrates these insights: The solid red line depicts the separating quantity for the case of $\beta = 0$; in this case, the separating quantity (weakly) increases in Δ. In contrast to this case, the dashed blue line depicts the separating quantity for $\beta = 0.2$. In this case, the separating quantity first increases in Δ; however, when the gap between the two possible states is considerably high, this trend is reversed.

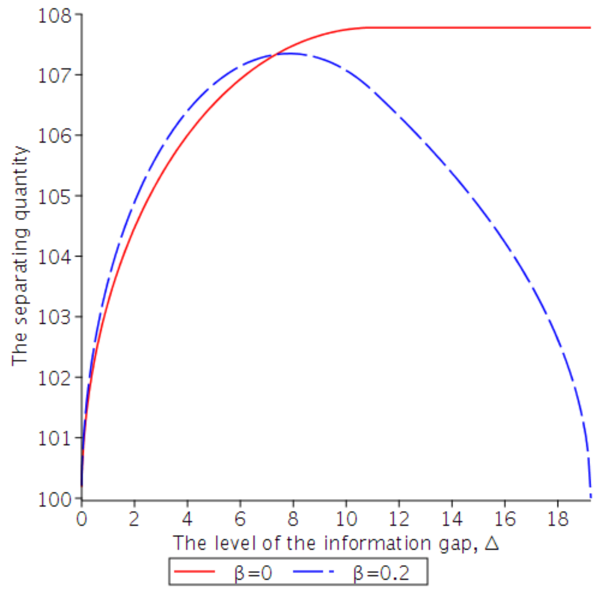

Figure D1. The separating quantity as a function of Δ for ε~U(-10,10), c=11, w=20, r=40 and μ_L=110.

The effect of PVO on consumers

Here, we study the effect of PVO on consumers, where we measure this effect in terms of the service level type II (also known as the *fill rate*) provided to consumers as a function of the level of financial holdings β. Service level type II is a quantity-oriented performance measure describing the proportion of the total demand that is satisfied based on the current capacity level, i.e., $1-\frac{\text{Expected unmet demand}}{\text{Expected demand}}$. In our case, the service level type II is calculated by:

$$SL_{II} = p\left(1-\frac{1}{\mu_H}\int_{K_H-\mu_H}^{\bar{\varepsilon}} (\mu_H+\varepsilon-K_H)f(\varepsilon)d\varepsilon\right)+(1-p)\left(1-\frac{1}{\mu_L}\int_{K_L-\mu_L}^{\bar{\varepsilon}} (\mu_L+\varepsilon-K_L)f(\varepsilon)d\varepsilon\right), \quad \text{(D.1)}$$

where K_L and K_H are set by the supplier according to the equilibrium. The first (second) element in the expression denotes the service level type II when demand is high (low). Note that the capacity positively affects this performance measure. Therefore, we obtain the following result:

Proposition D2. ***PVO and service level***

The effect of PVO on the service level is as follows:

(i) When $\beta \geq \underline{\beta}$, the service level increases with β.

(ii) When cheap talk or the separating equilibrium is played, the service level is independent of β, and it is the same for both equilibrium types.

(iii) When the pooling equilibrium is played, the service level is independent of β *(and it is different from that in case (ii)).*

The results in Proposition D2 can be explained as follows. When $\beta \geq \underline{\beta}$, the retailer finances the entire capacity in the market; this capacity level increases in β (see Theorem 1), which means that consumers benefit from a higher service level. When $\underline{\underline{\beta}} \leq \beta < \underline{\beta}$, information is exchanged via cheap talk. In this case, the capacity in the market is determined by the supplier's economic factors and it is independent of the financial holdings level. When $\beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\}$, we have shown that there are two possible outcomes: a separating equilibrium and a pooling equilibrium. In both of these cases, the market capacity is determined by the supplier, and it is independent of the financial holdings level. Note that this capacity level (and thus the service level) differs between the two types of equilibrium: in the separating equilibrium, the capacity is determined based on the realized market state (high or low), whereas in the pooling equilibrium it is determined based on the supplier's prior belief.

Appendix E. Proofs

Proof of equations (2)-(4)

Equation (2) holds because

$$E\left[\min(K,\max(\mu_i+\varepsilon,q^{adv}))\right]=q^{adv}F(q^{adv}-\mu_i)+\int_{q^{adv}-\mu_i}^{K-\mu_i}(\mu_i+x)f(x)dx+K(1-F(K-\mu_i))$$

$$=q^{adv}F(q^{adv}-\mu_i)+\mu_i(F(K-\mu_i)-F(q^{adv}-\mu_i))+(K-\mu_i)F(K-\mu_i)-(q^{adv}-\mu_i)F(q^{adv}-\mu_i)\,.$$

$$-\int_{q^{adv}-\mu_i}^{K-\mu_i}F(x)dx+K(1-F(K-\mu_i))=K-\int_{q^{adv}-\mu_i}^{K-\mu_i}F(x)dx$$

Equation (3) holds because

$$E\left[\min(\mu_i+\varepsilon,K)\right]=\int_{\underline{\varepsilon}}^{K-\mu_i}(\mu_i+x)f(x)dx+K(1-F(K-\mu_i))$$

$$=\mu_iF(K-\mu_i)+(K-\mu_i)F(K-\mu_i)-\int_{\underline{\varepsilon}}^{K-\mu_i}F(x)dx+K(1-F(K-\mu_i))=K-\int_{\underline{\varepsilon}}^{K-\mu_i}F(x)dx$$

and

$$E\left[\max(q^{adv}-\mu_i-\varepsilon,0)\right]=\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i}(q^{adv}-\mu_i-x)f(x)dx=(q^{adv}-\mu_i)F(q^{adv}-\mu_i)-\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i}xf(x)dx$$

$$=(q^{adv}-\mu_i)F(q^{adv}-\mu_i)-(q^{adv}-\mu_i)F(q^{adv}-\mu_i)+\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i}F(x)dx=\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i}F(x)dx$$

Equation (4) is based on the following derivation:

$$v_r(q^{adv},K\,|\,\mu_i)=\pi_r(q^{adv},K\,|\,\mu_i)+\beta\pi_s(K\,|\,\mu_i,q^{adv})$$

$$=(r-w)\left(K-\int_{\underline{\varepsilon}}^{K-\mu_i}F(x)dx\right)-w\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i}F(x)dx+\beta w\left(z_sK-\int_{q^{adv}-\mu_i}^{K-\mu_i}F(x)dx\right)$$

$$=r\left(K-\int_{\underline{\varepsilon}}^{K-\mu_i}F(x)dx\right)-w\left(K-\int_{q^{adv}-\mu_i}^{K-\mu_i}(1-\overline{F}(x))dx\right)+\beta w\left(z_sK-\int_{q^{adv}-\mu_i}^{K-\mu_i}F(x)dx\right)$$

$$=r\left(\mu_i+\underline{\varepsilon}+\int_{\underline{\varepsilon}}^{K-\mu_i}\overline{F}(x)dx\right)-\left(w(1-\beta)+\beta c\right)\left(q^{adv}+\int_{q^{adv}-\mu_i}^{K-\mu_i}\overline{F}(x)dx\right)-\beta c\left(K-q^{adv}-\int_{q^{adv}-\mu_i}^{K-\mu_i}\overline{F}(x)dx\right)$$

$$=r\left(\mu_i+\underline{\varepsilon}+\int_{\underline{\varepsilon}}^{K-\mu_i}\overline{F}(x)dx\right)-w(1-\beta)\left(q^{adv}+\int_{q^{adv}-\mu_i}^{K-\mu_i}\overline{F}(x)dx\right)-\beta cK$$

Proof of Theorem 1

Let $G(\beta) \equiv \frac{1}{r}\left(v_r(q^*, q^* \mid \mu_i) - v_r(0, \mu_i + F^{-1}(z_s) \mid \mu_i)\right)$ be the retailer's accounting payoff gain (measured in multiples of the retail price) from placing an optimal advance order q^* rather than being satisfied with the capacity the supplier offers for free $(\mu_i + F^{-1}(z_s))$. In what follows, we develop the explicit formula for $G(\beta)$, and we reveal certain properties of the formula that are necessary to prove the theorem. By applying equations (2)-(4), with $K = q^{adv}$, we obtain

$$
\begin{aligned}
v_r(q^{adv}, q^{adv} \mid \mu_i) &= \pi_r(q^{adv}, q^{adv} \mid \mu_i) + \beta \pi_s(q^{adv} \mid \mu_i, q^{adv}) \\
&= (r-w)\left(q^{adv} - \int_{\underline{\varepsilon}}^{q^{adv}-\mu_i} F(x)dx\right) - w\int_{\underline{\varepsilon}}^{q^{adv}-\mu_i} F(x)dx + \beta w z_s q^{adv} \\
&= r\left(q^{adv}\left(1-(1-z_r)(1-\beta z_s)\right) - \int_{\underline{\varepsilon}}^{q^{adv}-\mu_i} F(x)dx\right) = r\left(q^{adv} z_r(\beta) - \int_{\underline{\varepsilon}}^{q^{adv}-\mu_i} F(x)dx\right).
\end{aligned}
$$

We then calculate the first and second derivatives of the retailer's payoff with respect to the advance order to obtain

$$
\frac{\partial v_r(q^{adv}, q^{adv} \mid \mu_i)}{\partial q^{adv}} = r\left[z_r(\beta) - F(q^{adv} - \mu_i)\right] \text{ and } \frac{\partial^2 v_r(q^{adv}, q^{adv} \mid \mu_i)}{\partial (q^{adv})^2} = -rf(q^{adv} - \mu_i) < 0 .
$$

Thus, the retailer's payoff when she finances the entire capacity in the market is a concave function of q^{adv} with a maximum at $q^* = \mu_i + F^{-1}\left(z_r(\beta)\right)$ and a value of

$$
v_r(q^*, q^* \mid \mu_i) = r\left[z_r(\beta)\left(\mu_i + F^{-1}(z_r(\beta))\right) - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx\right].
$$

When the retailer is satisfied with the capacity the supplier secures for free, her payoff is

$$
\begin{aligned}
v_r(0, \mu_i + F^{-1}(z_s) \mid \mu_i) &= \pi_r(0, \mu_i + F^{-1}(z_s) \mid \mu_i) + \beta \pi_s(\mu_i + F^{-1}(z_s) \mid \mu_i, 0) \\
&= (r-w)\left(\mu_i + F^{-1}(z_s) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right) + \beta w\left(z_s(\mu_i + F^{-1}(z_s)) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right) \\
&= r\left[z_r\left(\mu_i + F^{-1}(z_s) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right) + \beta(1-z_r)\left(z_s(\mu_i + F^{-1}(z_s)) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right)\right] \\
&= r\left[\left(z_r + \beta z_s(1-z_r)\right)\left(\mu_i + F^{-1}(z_s)\right) - \left(z_r + \beta(1-z_r)\right)\int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right] \\
&= r\left[z_r(\beta)\left(\mu_i + F^{-1}(z_s)\right) - \left(z_r + \beta(1-z_r)\right)\int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right].
\end{aligned}
$$

Therefore, $G(\beta)=z_r(\beta)\left(F^{-1}(z_r(\beta))-F^{-1}(z_s)\right)+\left(z_r+\beta(1-z_r)\right)\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx-\int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))}F(x)dx$.

Lemma E1. Under complete information, a unique threshold value $\underline{\beta}\in[0,1)$ exists, such that $G(\beta)\geq 0$ if and only if $\beta\geq\underline{\beta}$.

Proof. The absence of partial ownership (i.e., $\beta=0$) results in $z_r(0)=z_r$, which implies that

$$G(0)=z_r\left(F^{-1}(z_r)-F^{-1}(z_s)+\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx\right)-\int_{\underline{\varepsilon}}^{F^{-1}(z_r)}F(x)dx,$$

where $G(0)$ can be either positive or negative according to the values of z_r and z_s.

Complete ownership (i.e., $\beta=1$) results in $z_r(1)=z_c\equiv 1-c/r$ (the critical fractile of a centralized supply chain), which implies that

$$\begin{aligned}G(1)&=z_c\left(F^{-1}(z_c)-F^{-1}(z_s)\right)-\int_{F^{-1}(z_s)}^{F^{-1}(z_c)}F(x)dx>z_c\left(F^{-1}(z_c)-F^{-1}(z_s)\right)-\int_{F^{-1}(z_s)}^{F^{-1}(z_c)}F(F^{-1}(z_c))dx\\&=z_c\left(F^{-1}(z_c)-F^{-1}(z_s)\right)-\int_{F^{-1}(z_s)}^{F^{-1}(z_c)}z_c dx=0\end{aligned}$$

Differentiating $G(\beta)$, and using the relations $\frac{dz_r(\beta)}{d\beta}=z_s(1-z_r)$ and $\frac{d}{d\beta}\int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))}F(x)dx=z_r(\beta)\frac{dF^{-1}(z_r(\beta))}{d\beta}$, we obtain $G'(\beta)=(1-z_r)\left(z_s\left(F^{-1}(z_r(\beta))-F^{-1}(z_s)\right)+\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx\right)$. A necessary condition for incentivizing the retailer to order in advance is $z_r(\beta)>z_s$ because otherwise, the supplier builds a higher capacity for free than the retailer's optimal capacity. Thus, it is clear that the domain of β for which $G(\beta)>0$ is restricted to values of β that satisfy $z_r(\beta)>z_s$, and that in this domain, $G'(\beta)>0$. Hence, and since $G(1)>0$, a unique value $\underline{\beta}\in[0,1)$ exists such that $G(\beta)\geq 0$ if and only if $\beta\geq\underline{\beta}$. ■

By Lemma E1, as the retailer's partial ownership increases (i.e., for larger values of β), from a certain threshold, denoted by $\underline{\beta}$, between zero ownership and full ownership, placing an optimal advance order is more beneficial for the retailer than being satisfied with the capacity the supplier secures for free (i.e., $v_r(q^*,q^*\mid\mu_i)\geq v_r(0,\mu_i+F^{-1}(z_s)\mid\mu_i)$ if and only if $\beta\geq\underline{\beta}$).

Proof of Theorem 2

(i) We first show that Condition (12) is always satisfied. Recall that $v_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_H) = \pi_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_H) + \beta\pi_s(\mu_H + F^{-1}(z_s)\,|\,\mu_H,0)$. Since the retailer's operational profit increases with the capacity level the supplier prepares for free, $\pi_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_H) > \pi_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_H)$. In addition, since $\mu_H + F^{-1}(z_s)$ maximizes the supplier's payoff when no advance order is placed and demand is high, $\pi_s(\mu_H + F^{-1}(z_s)\,|\,\mu_H,0) > \pi_s(\mu_L + F^{-1}(z_s)\,|\,\mu_H,0)$. Therefore, we obtain

$$\begin{aligned} v_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_H) &= \pi_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_H) + \beta\pi_s(\mu_H + F^{-1}(z_s)\,|\,\mu_H,0) \\ &> \pi_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_H) + \beta\pi_s(\mu_L + F^{-1}(z_s)\,|\,\mu_H,0) = v_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_H), \end{aligned}$$

which concludes the proof that Condition (12) is always satisfied. Condition (13) is satisfied when

$$\begin{aligned} v_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_L) &= \pi_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_L) + \beta\pi_s(\mu_L + F^{-1}(z_s)\,|\,\mu_L,0) \\ &\geq \pi_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_L) + \beta\pi_s(\mu_H + F^{-1}(z_s)\,|\,\mu_L,0) = v_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_L). \end{aligned}$$

Simple manipulation shows that this condition is met when

$$\beta \geq \underline{\underline{\beta}} \equiv \frac{\pi_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_L) - \pi_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_L)}{\pi_s(\mu_L + F^{-1}(z_s)\,|\,\mu_L,0) - \pi_s(\mu_H + F^{-1}(z_s)\,|\,\mu_L,0)}. \tag{E.1}$$

Finally, note that both the numerator, which captures the retailer's gain from pretending to be a high type, and the denominator, which captures the supplier's gain from disbelieving such pretense, are positive. Using (2) and (3), the four expected profits that comprise the expression for $\underline{\underline{\beta}}$ in (E.1) can be written as:

$$\pi_r(0,\mu_H + F^{-1}(z_s)\,|\,\mu_L) = rz_r\left(\mu_H + F^{-1}(z_s) - \int_{\underline{\varepsilon}}^{\mu_H-\mu_L+F^{-1}(z_s)} F(x)dx\right),$$

$$\pi_r(0,\mu_L + F^{-1}(z_s)\,|\,\mu_L) = rz_r\left(\mu_L + F^{-1}(z_s) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right),$$

$$\pi_s(\mu_L + F^{-1}(z_s)\,|\,\mu_L,0) = w\left(z_s(\mu_L + F^{-1}(z_s)) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx\right),$$

$$\pi_s(\mu_H + F^{-1}(z_s)\,|\,\mu_L,0) = w\left(z_s(\mu_H + F^{-1}(z_s)) - \int_{\underline{\varepsilon}}^{\mu_H-\mu_L+F^{-1}(z_s)} F(x)dx\right).$$

By substituting the above four expressions into (B.1), and recalling that $\Delta \equiv \mu_H - \mu_L$ and $R(\Delta) \equiv \frac{1}{\Delta}\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(x)dx$, we obtain

$$\underline{\underline{\beta}} = \frac{z_r}{1-z_r}\left(\frac{1-z_s}{R(\Delta)-z_s}-1\right). \tag{E.2}$$

In order to find the range of $\underline{\underline{\beta}}$, we analyze $R(\Delta)$. Using L'Hôpital's rule,

$$\lim_{\Delta\to 0} R(\Delta) = \lim_{\Delta\to 0}\frac{1}{\Delta}\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(x)dx = \lim_{\Delta\to 0} F(\Delta + F^{-1}(z_s)) = z_s,$$

$$\lim_{\Delta\to \infty} R(\Delta) = \lim_{\Delta\to \infty}\frac{1}{\Delta}\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(x)dx = \lim_{\Delta\to \infty} F(\Delta + F^{-1}(z_s)) = 1.$$

By differentiation,

$$\frac{\partial R(\Delta)}{\partial \Delta} = \frac{\Delta F(\Delta + F^{-1}(z_s)) - \int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(x)dx}{\Delta^2} > \frac{\Delta F(\Delta + F^{-1}(z_s)) - \int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(\Delta + F^{-1}(z_s))dx}{\Delta^2} = 0.$$

Thus, $R(\Delta)$ is a monotonically increasing function defined over the range $(z_s, 1)$. By (E.2), $\underline{\underline{\beta}}$ decreases in $R(\Delta)$, so it also decreases in Δ from $\lim_{\Delta\to 0}\underline{\underline{\beta}} = \infty$ to $\lim_{\Delta\to\infty}\underline{\underline{\beta}} = 0$. Hence, by combining (E.1) and (E.2), the condition for meaningful cheap-talk information exchange can be written as $\beta \geq \frac{z_r}{1-z_r}\left(\frac{1-z_s}{R(\Delta)-z_s}-1\right)$.

(ii) Using algebraic manipulations, the condition derived at the end of the proof of Theorem 2(i) can be written as $R(\Delta) \geq z_s + \frac{1-z_s}{1+\beta(1/z_r - 1)}$. Because $R(\Delta)$ is a monotonically increasing function, its inverse function, $R^{-1}(\cdot)$, exists, and the condition can be written as $\Delta > \underline{\underline{\Delta}} \equiv R^{-1}\left(z_s + \frac{1-z_s}{1+\beta(1/z_r - 1)}\right)$.

Proof of Proposition 1

According to Theorem 2(i), cheap talk is possible when

$$\underline{\underline{\beta}} \equiv \frac{\pi_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L) - \pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L)}{\pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0) - \pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, 0)} < 1.$$

This condition can be written as

$$\begin{aligned}\pi_{sc}(\mu_H + F^{-1}(z_s) \mid \mu_L) &= \pi_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L) + \pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, 0)\\ &< \pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L) + \pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0) = \pi_{sc}(\mu_L + F^{-1}(z_s) \mid \mu_L).\end{aligned}$$

Proof of Theorem 3

(i) The condition for achieving an efficient separation that satisfies the intuitive criterion (Cho and Kreps 1987) is $v_r(0,\mu_L+F^{-1}(z_s)\,|\,\mu_L)=v_r(\tilde q,\mu_H+F^{-1}(z_s)\,|\,\mu_L)$. Using (2)-(4), it can be written as follows:

$$(r-w)\left(\mu_L+F^{-1}(z_s)-\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx\right)+\beta w\left(z_s(\mu_L+F^{-1}(z_s))-\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx\right)$$

$$=(r-w)\left(\mu_H+F^{-1}(z_s)-\int_{\underline{\varepsilon}}^{\Delta+F^{-1}(z_s)}F(x)dx\right)-w\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx+\beta w\left(z_s(\mu_H+F^{-1}(z_s))-\int_{\tilde q-\mu_L}^{\Delta+F^{-1}(z_s)}F(x)dx\right).$$

Using algebraic manipulations, the latter equation can be reformulated as

$$(r-w)\left(\Delta-\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)}F(x)dx\right)-w\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx+\beta w\left(z_s\Delta+\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx-\int_{\tilde q-\mu_L}^{\Delta+F^{-1}(z_s)}F(x)dx\right)=0. \quad \text{(E.3)}$$

Dividing (E.3) by *w*, and using the relations $z_r=1-\dfrac{w}{r}$ and $\displaystyle\int_{\tilde q-\mu_L}^{\Delta+F^{-1}(z_s)}F(x)dx=\int_{\underline{\varepsilon}}^{\Delta+F^{-1}(z_s)}F(x)dx-\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx,$

results in

$$\frac{z_r}{1-z_r}\left(\Delta-\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)}F(x)dx\right)-\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx+\beta\left(z_s\Delta-\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)}F(x)dx+\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx\right)=0,$$

from which, using the definition $R(\Delta)\equiv\dfrac{1}{\Delta}\displaystyle\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)}F(x)dx$, we extract the relation:

$$\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx=\frac{\Delta}{1-\beta}\left[\frac{z_r}{1-z_r}\big(1-R(\Delta)\big)+\beta\big(z_s-R(\Delta)\big)\right]. \quad \text{(E.4)}$$

Using algebraic manipulations, (E.4) can be written as

$$\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx=\frac{\Delta\left[z_r(1-z_s)-\left(z_r+\beta(1-z_r)\right)\left(R(\Delta)-z_s\right)\right]}{(1-\beta)(1-z_r)}. \quad \text{(E.5)}$$

From Theorem 2(i), we extract $R(\Delta)=z_s+\dfrac{1-z_s}{1+\underline{\underline{\beta}}(1/z_r-1)}$ and substitute it into (E.5) to obtain

$$\int_{\underline{\varepsilon}}^{\tilde q-\mu_L}F(x)dx=\frac{\Delta(1-z_s)}{1+\underline{\underline{\beta}}(1/z_r-1)}\frac{\underline{\underline{\beta}}-\beta}{1-\beta}.$$

The claim is proved using the relation $(\underline{\underline{\beta}}-\beta)/(1-\beta)=1-(1-\underline{\underline{\beta}})/(1-\beta)$.

(ii) Straightforward from the strategy of the supplier.

Proof of Proposition 2

Implicit differentiation of (17) with respect to β yields $\frac{d\tilde{q}}{d\beta} = \frac{1}{F(\tilde{q}-\mu_L)} \cdot \frac{\Delta(1-z_s)}{1+\underline{\underline{\beta}}(1/z_r-1)} \cdot \frac{\underline{\underline{\beta}}-1}{(1-\beta)^2}$. The claim is proved since $\operatorname{sgn}\left(\frac{d\tilde{q}}{d\beta}\right) = \operatorname{sgn}\left(\underline{\underline{\beta}}-1\right)$.

Proof of Proposition 3

By applying (2),

$$\begin{aligned}
&\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, \tilde{q}_0) - \pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0) \\
&= w\left(z_s(\mu_H + F^{-1}(z_s)) - \int_{\tilde{q}_0-\mu_L}^{\Delta+F^{-1}(z_s)} F(x)dx \right) - w\left(z_s(\mu_L + F^{-1}(z_s)) - \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx \right) \\
&= w\left(z_s\Delta - \int_{\tilde{q}_0-\mu_L}^{\Delta+F^{-1}(z_s)} F(x)dx + \int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx \right) = w\left(z_s\Delta - \int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(x)dx + \int_{\underline{\varepsilon}}^{\tilde{q}_0-\mu_L} F(x)dx \right) \\
&= w\Delta\left(z_s - R(\Delta) + \frac{1}{\Delta}\int_{\underline{\varepsilon}}^{\tilde{q}_0-\mu_L} F(x)dx \right).
\end{aligned}$$

By (17), the separating quantity when $\beta = 0$, denoted by $\tilde{q}_0$, satisfies $\int_{\underline{\varepsilon}}^{\tilde{q}_0-\mu_L} F(x)dx = \frac{\Delta(1-z_s)\underline{\underline{\beta}}}{1+\underline{\underline{\beta}}(1/z_r-1)}$, and by Theorem 2(i), $R(\Delta) = \frac{1-z_s}{1+\underline{\underline{\beta}}(1/z_r-1)} + z_s$; thus

$$\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, \tilde{q}_0) - \pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0) = \frac{w\Delta(1-z_s)(\underline{\underline{\beta}}-1)}{1+\underline{\underline{\beta}}(1/z_r-1)},$$

where the sign of the left-hand side equals the sign of $\underline{\underline{\beta}}-1$. Following Proposition 2, the claim is proved.

Proof of Theorem 4

We start by stating the behavior of the two retailer types under the pooling equilibrium in the following lemma.

Lemma E2. If a pooling equilibrium exists, both retailer types prefer to order in advance a quantity of $q^P = 0$.

Proof. First, let us assume that in the pooling equilibrium, both retailer types indeed order zero units in advance. In this case, the supplier's optimal capacity is:

$$K^P = \arg\max_K \left\{ \pi_s = pw\left(z_s K - \int_{\underline{\varepsilon}}^{K-\mu_H} F(x)dx \right) + (1-p)w\left(z_s K - \int_{\underline{\varepsilon}}^{K-\mu_L} F(x)dx \right) \right\}. \quad \text{(E.6)}$$

Since $\frac{d^2\pi_s}{dK^2} = -w\left(pf(K-\mu_H)+(1-p)f(K-\mu_L)\right)<0$, π_s is concave in *K*, implying that K^P is the unique solution of the necessary condition: $pF(K-\mu_H)+(1-p)F(K-\mu_L)=z_s$.

Now, assume to the contrary that there exists another efficient pooling equilibrium in which both retailer types order a quantity $\hat{q}$. If $\hat{q}\in(0,K^P)$, then the supplier would still build the capacity K^P. When the retailer orders $\hat{q}$, her payoff is given by $v_r(\hat{q},K^P\mid\mu_i)$. It is clear that $v_r(\hat{q},K^P\mid\mu_i)$ decreases in $\hat{q}$ over the domain $[0,K^P)$, because the retailer orders more units in advance but receives the same capacity level. Thus, for any pooling equilibrium that results in capacity K^P, the retailer would prefer the one in which $q^P=0$.

Let us assume by contradiction that there is a pooling equilibrium in which $\hat{q}\geq K^P$. Then the supplier will build a capacity that matches the retailer's advance-order quantity, so the retailer actually finances the entire capacity in the market. In such a case, each retailer type would prefer to finance the capacity determined by her economic factors, and, consequently, no pooling equilibrium can exist. ■

We next show that for the belief system outlined in Theorem 4, any deviation from the proposed strategy is unprofitable. First, we show that ordering a positive number of units in advance that is lower than $\overline{q}^P$ (given in (19)) results in the same belief of the supplier as ordering $q^P=0$, and thus the same capacity level will be secured. Since $v_r(\hat{q},K^P\mid\mu_i)$ decreases in $\hat{q}$, such a deviation will not be profitable.

Second, consider a deviation in which the high-type retailer orders an advance quantity $\hat{q}\in\left[\overline{q}^P,\mu_H+F^{-1}(z_s)\right]$. Based on the belief system outlined in the theorem, the supplier believes that demand is high and builds the capacity level $\mu_H+F^{-1}(z_s)$. Accordingly, the corresponding payoff of the high-type retailer is

$$\begin{aligned} v_r(\hat{q},\mu_H+F^{-1}(z_s)\mid\mu_H) &= \pi_r(\hat{q},\mu_H+F^{-1}(z_s)\mid\mu_H)+\beta\pi_s(\mu_H+F^{-1}(z_s)\mid\mu_H,\hat{q}) \\ &= (r-w(1-\beta z_s))(\mu_H+F^{-1}(z_s))-(r-w(1-\beta))\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx-w(1-\beta)\int_{\underline{\varepsilon}}^{\hat{q}-\mu_H}F(x)dx, \end{aligned} \tag{E.7}$$

which is a decreasing function of $\hat{q}$. Under the suggested pooling equilibrium, the payoff of the high-type retailer is given by:

$$v_r(0,K^P\mid\mu_H)=\pi_r(0,K^P\mid\mu_H)+\beta\pi_s(K^P\mid\mu_H,0)=(r-w(1-\beta z_s))K^P-(r-w(1-\beta))\int_{\underline{\varepsilon}}^{K^P-\mu_H}F(x)dx. \tag{E.8}$$

Now, we show that by ordering $\hat{q}=\overline{q}^P$, the high-type retailer is indifferent between following the suggested pooling equilibrium and deviating from it. Using equation (19), from which $\overline{q}^P$ is extracted, i.e.,

$$\int_{\underline{\varepsilon}}^{\overline{q}^P-\mu_H} F(x)dx = \frac{1}{w(1-\beta)}\left[(r-w(1-\beta z_s))(\mu_H + F^{-1}(z_s) - K^p) - (r-w(1-\beta))\int_{K^p-\mu_H}^{F^{-1}(z_s)} F(x)dx\right],$$

we obtain the following relationship:

$$v_r(\overline{q}^P, \mu_H + F^{-1}(z_s) \mid \mu_H) = (r-w(1-\beta z_s))(\mu_H + F^{-1}(z_s)) - (r-w(1-\beta))\int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx$$

$$-\left((r-w(1-\beta z_s))(\mu_H + F^{-1}(z_s) - K^p) - (r-w(1-\beta))\int_{K^p-\mu_H}^{F^{-1}(z_s)} F(x)dx\right)$$

$$= r - w(1-\beta z_s))K^p - (r-w(1-\beta))\int_{\underline{\varepsilon}}^{K^p-\mu_H} F(x)dx = v_r(0, K^P \mid \mu_H).$$

The retailer will not wish to deviate from the proposed pooling equilibrium for $\hat{q} \in \left[\overline{q}^P, \mu_H + F^{-1}(z_s)\right]$ when $v_r(0, K^P \mid \mu_H) \geq v_r(\hat{q}, \mu_H + F^{-1}(z_s) \mid \mu_H)$. Since $v_r(\hat{q}, \mu_H + F^{-1}(z_s) \mid \mu_H)$ is a decreasing function of $\hat{q}$, any deviation to $\hat{q} \in \left[\overline{q}^P, \mu_H + F^{-1}(z_s)\right]$ will not be profitable.

Finally, consider a deviation in which the high-type retailer orders an advance quantity $\hat{q} > \mu_H + F^{-1}(z_s)$. Such a deviation results in the supplier's belief that demand is high (since $\hat{q} > \overline{q}^P$). However, since the ordered quantity is higher than the supplier's optimal capacity for this belief, the supplier simply builds a capacity to match this order. Note also that such an order quantity implies that the retailer finances the entire capacity in the market. Consequently, a pooling equilibrium cannot exist, since the low-type retailer prefers not to mimic the advance order of the high-type retailer in this case. Therefore, we now characterize the condition under which a deviation to quantity $\hat{q} > \mu_H + F^{-1}(z_s)$ by the high-type retailer would be profitable, which would imply that a pooling equilibrium cannot exist. A high-type retailer would prefer to finance the entire capacity, rather than to accept the pooling equilibrium, if the following condition were satisfied: $v_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H) \geq v_r(0, K^P \mid \mu_H)$. In what follows, we show that there is a threshold value of β, denoted by β^P, above which the high-type retailer is better off with financing the entire capacity in the market rather than accepting the capacity built by the supplier under the pooling equilibrium.

Lemma E3. K^P is an increasing function of *p*.

Proof. Taking the implicit derivative of the equation from which K^P is extracted (given in (18)) with respect to *p* yields: $\dfrac{dK^P}{dp} = \dfrac{F(K^P - \mu_L) - F(K^P - \mu_H)}{pf(K^P - \mu_H) + (1-p)f(K^P - \mu_L)} > 0$. ■

By Lemma E3, K^P is maximized for $p=1$. Therefore, its maximal value is calculated by solving $F(K^P - \mu_H) = z_s$, which results in $K^P = \mu_H + F^{-1}(z_s)$.

Similar to the proof of Theorem 1, herein, we define

$$G^P(\beta) \equiv \frac{1}{r}\left(v_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H) - v_r(0, K^P \mid \mu_H)\right)$$

as the high-type retailer's accounting payoff gain (measured in multiples of the retail price) from placing an optimal advance order in the case where she finances the entire capacity in the market instead of being satisfied with the capacity the supplier offers for free under the prior belief (K^P). By (2)-(4), we have

$$v_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H) = r\left(\left(\mu_H + F^{-1}(z_r(\beta)\right) z_r(\beta) - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx\right),$$

and

$$v_r(0, K^P \mid \mu_H) = r\left[K^P z_r(\beta) - \left(z_r + \beta(1-z_r)\right) \int_{\underline{\varepsilon}}^{K^P - \mu_H} F(x)dx\right].$$

Thus, $G^P(\beta) = z_r(\beta)\left(F^{-1}(z_r(\beta)) - (K^P - \mu_H)\right) + \left(z_r + \beta(1-z_r)\right) \int_{\underline{\varepsilon}}^{K^P - \mu_H} F(x)dx - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx$.

Lemma E4. There is a unique threshold value $\beta^P \in [0,1)$ such that $G^P(\beta) \geq 0$ (i.e., $v_r(0, K^P \mid \mu_H) \leq v_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H)$) if and only if $\beta \geq \beta^P$.

Proof. Similar to the proof of Theorem 1, the absence of partial ownership (i.e., $\beta = 0$) results in

$$G^P(0) = z_r\left(F^{-1}(z_r) - (K^P - \mu_H) + \int_{\underline{\varepsilon}}^{K^P - \mu_H} F(x)dx\right) - \int_{\underline{\varepsilon}}^{F^{-1}(z_r)} F(x)dx,$$

where $z_r \equiv z_r(0)$ and $G^P(0)$ can be either positive or negative.

Complete ownership (i.e., $\beta = 1$) results in

$$G^P(1) = z_c\left(F^{-1}(z_c) - (K^p - \mu_H)\right) - \int_{K^P - \mu_H}^{F^{-1}(z_c)} F(x)dx > z_c\left(F^{-1}(z_c) - (K^P - \mu_H)\right) - \int_{K^P - \mu_H}^{F^{-1}(z_c)} F(F^{-1}(z_c))dx$$

$$= z_c\left(F^{-1}(z_c) - (K^P - \mu_H)\right) - \int_{K^P - \mu_H}^{F^{-1}(z_c)} z_c dx = 0,$$

where $z_c \equiv z_r(1) = 1 - c/r > z_s = 1 - w/r$. Using equation (18), it is easy to show that:

$$F^{-1}(z_c) > F^{-1}(z_s) = F^{-1}(pF(K^P - \mu_H) + (1-p)F(K^P - \mu_L))$$
$$> F^{-1}(pF(K^P - \mu_H) + (1-p)F(K^P - \mu_H)) > F^{-1}(F(K^P - \mu_H)) = K^P - \mu_H.$$

Differentiating $G^P(\beta)$ and using the relations $\frac{dz_r(\beta)}{d\beta} = z_s(1-z_r)$ and $\frac{d}{d\beta}\int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx = z_r(\beta)\frac{dF^{-1}(z_r(\beta))}{d\beta}$, we obtain $G^P{}'(\beta) = (1-z_r)\left(z_s\left(F^{-1}(z_r(\beta)) - (K^P - \mu_H)\right) + \int_{\underline{\varepsilon}}^{K^P-\mu_H} F(x)dx \right)$. A necessary condition for incentivizing the high-type retailer to finance the entire capacity in the market is $\mu_H + F^{-1}(z_r(\beta)) > K^P$, because otherwise, the supplier builds a higher capacity for free than the retailer's optimal capacity. Thus, it is clear that the domain of β for which $G^P(\beta) > 0$ is restricted to values of β which satisfy $\mu_H + F^{-1}(z_r(\beta)) > K^P$, and that in this domain, $G^P{}'(\beta) > 0$. Hence, and considering also that $G^P(1) > 0$, a unique value $\beta^P \in [0,1)$ exists such that $G^P(\beta) \geq 0$ if and only if $\beta \geq \beta^P$. ■

By Lemma E4, as the high-type retailer's PVO level increases (i.e., a larger value of β), from a certain threshold, denoted by β^P, between zero ownership and full ownership, placing an optimal advance order is more beneficial to the retailer than being satisfied with the capacity the supplier secures for free (i.e., $v_r(\mu_H + F^{-1}(z_r(\beta^P)), \mu_H + F^{-1}(z_r(\beta^P)) \mid \mu_H) \geq v_r(0, K^P \mid \mu_H)$ if and only if $\beta \geq \beta^P$). By the definition of $G^P(\beta)$ and Lemma E4, we conclude that

$$\beta^P \equiv \min_{\beta\in[0,1]} \left\{ \beta \;\middle|\; \begin{array}{l} z_r(\beta)\left(F^{-1}(z_r(\beta)) - (K^P - \mu_H)\right) \\ +\left(z_r + \beta(1-z_r)\right) \int_{\underline{\varepsilon}}^{K^P-\mu_H} F(x)dx - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx \geq 0 \end{array} \right\} < 1$$

is the minimal value of β that ensures the high-type retailer prefers to finance the entire capacity.

The following lemma tightens the upper bound of β^P.

Lemma E5. $\beta^P < \underline{\beta}$

Proof. Let $\Upsilon_r(\beta) \equiv v_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H)$ be the maximal total payoff of a high-type retailer who holds a share of β in the supplier and secures the entire capacity in the market. According to the proof of Theorem 1, $\Upsilon_r(\beta) = r\left[z_r(\beta)\left(\mu_H + F^{-1}(z_r(\beta))\right) - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx \right]$; according to the proof of Lemma E4, if $\beta^P > 0$ then β^P is the solution of $\Upsilon_r(\beta) = v_r(0, K^P \mid \mu_H)$; and according to the proof of Theorem 1, if $\underline{\beta} > 0$ then $\underline{\beta}$ is the solution of

$\Upsilon_r(\beta) = v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_H)$. Differentiating $\Upsilon_r(\beta)$ with respect to β yields $\Upsilon_r'(\beta) = rz_s(1-z_r)\left(\mu_H + F^{-1}(z_r(\beta))\right) = (w-c)\left(\mu_H + F^{-1}(z_r(\beta))\right) > 0$ implying that $\Upsilon_r(\beta)$ is an increasing function of β. By the proof of Lemma E3, $K^P \le \mu_H + F^{-1}(z_s)$, so $v_r(0, K^P \mid \mu_H) \le v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_H)$. Hence, $\beta^P < \underline{\beta}$. ■

Proof of Proposition 4

(i) It was shown in Theorem 1 that, when $\beta \ge \underline{\beta}$, the retailer is not satisfied with the capacity that the supplier secures even under complete information. Furthermore, even when cheap talk is possible, under this condition ($\beta \ge \underline{\beta}$), the retailer prefers to finance the entire capacity. Note that if the retailer prefers financing the entire capacity over engaging in cheap talk, then she also prefers financing the entire capacity over playing the signaling game (which is more costly for the retailer than cheap talk). Furthermore, in this region ($\beta \ge \underline{\beta}$), financing the entire capacity in the market is also preferable to the pooling equilibrium since

$$v_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H) \ge v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_H) \ge v_r(0, K^P \mid \mu_H).$$

The first inequality holds because $\beta \ge \underline{\beta}$, a region in which the retailer prefers financing the entire capacity over cheap-talk information exchange. The second inequality holds because $\mu_H + F^{-1}(z_s) \mid \mu_H) \ge K^P$ (see the proof of Lemma E3).

(ii) When $\underline{\underline{\beta}} \le \beta < \underline{\beta}$, the retailer is able to credibly exchange information with the supplier via cheap talk. In this region, $F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H) \le v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_H)$ because, when $\beta \le \underline{\beta}$, it is better for the high-type retailer to exchange information via cheap talk than to finance the entire capacity (and there is no need for the retailer to engage in a signaling game). Finally, note that the high-type retailer prefers cheap-talk information exchange to the pooling equilibrium because $v_r(0, \mu_H + F^{-1}(z_s) \mid \mu_H) \ge v_r(0, K^P \mid \mu_H)$ (as explained in (i)).

(iii) (a) When $\beta^P \le \beta < \min(\underline{\underline{\beta}}, \underline{\beta})$, the pooling equilibrium is not feasible and thus the signaling equilibrium will be played.

(b) When $\beta < \min(\beta^P, \underline{\underline{\beta}}, \underline{\beta})$, the signaling and the pooling equilibrium are both feasible. By Lemma E3, the capacity in the pooling equilibrium increases in *p*. At the limit $(p \to 1)$, the supplier builds the same capacity level under the pooling equilibrium as under the separating equilibrium (see proof of Lemma E3), but without the need for signaling. Since $\lim_{p \to 0} K^P = \mu_L + F^{-1}(z_s)$ and $\lim_{p \to 1} K^P =$

$\mu_H + F^{-1}(z_s)$, then $\lim_{p\to 0} v_r(0, K^P \mid \mu_H) \le v_r(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H) \le \lim_{p\to 1} v_r(0, K^P \mid \mu_H)$. Therefore, due to the monotonicity of K^P with respect to *p* (Lemma E3), for any costly signaling equilibrium, there is a threshold value $\bar{p}$ such that for any $p \ge \bar{p}$, the pooling equilibrium would provide the high-type retailer with greater payoff. The value of $\bar{p}$ is extracted by solving the equation $v_r(0, K^P \mid \mu_H) = v_r(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H)$, where K^P is obtained by solving $pF(K-\mu_H) + (1-p)F(K-\mu_L) = z_s$.

Proof of Proposition 5

The outline of the proof is as follows. Let $\underline{\beta}(w)$ and $\underline{\underline{\beta}}(w)$ be the values of $\underline{\beta}$ and $\underline{\underline{\beta}}$, respectively, as a function of *w*, both defined over the domain (c, r). First, we show that $\underline{\beta}(w)$ is an increasing function with range (0,1). We then show that $\underline{\underline{\beta}}(w)$ is either higher than 1, or, when $\underline{\underline{\beta}}(w) < 1$, it is a decreasing function with $\lim_{w\to r^-} \underline{\underline{\beta}}(w) = 0$. Hence, it is clear that there exists a value $w_0 \in (c, r)$ such that $\underline{\beta}(w_0) = \underline{\underline{\beta}}(w_0) = \beta_0$, $\underline{\beta}(w) < \underline{\underline{\beta}}(w)$ for $w \in (c, w_0)$, and $\underline{\beta}(w) > \underline{\underline{\beta}}(w)$ for $w \in (w_0, r)$. Let w_1 and w_2 be values of *w* such that $\underline{\beta}(w_1) = \beta$ and $\underline{\underline{\beta}}(w_2) = \beta$. Thus, we conclude that for $\beta < \beta_0$, $w_1 < w_2$, for $\beta = \beta_0$, $w_1 = w_2$, and for $\beta > \beta_0$, $w_1 > w_2$.

(i) When $w_1 < w_2$, for any $w \in (c, w_1)$, $\beta > \underline{\beta}(w)$, implying that the retailer finances the entire capacity; for any $w \in (w_1, w_2)$, $\beta < \min\{\underline{\beta}(w), \underline{\underline{\beta}}(w)\}$, implying that the separating or pooling equilibrium will be played; and for any $w \in (w_2, r)$, $\underline{\underline{\beta}}(w) < \beta < \underline{\beta}(w)$, implying that cheap talk will be used to exchange information.

(ii) When $w_1 = w_2 = w_0$, for any $w \in (c, w_0)$, $\beta > \underline{\beta}(w)$, implying that the retailer finances the entire capacity; and for any $w \in (w_0, r)$, $\underline{\underline{\beta}}(w) < \beta < \underline{\beta}(w)$, implying that cheap talk will be used to exchange information.

(iii) When $w_1 > w_2$, for any $w \in (c, w_1)$, $\beta > \underline{\beta}(w)$, implying that the retailer finances the entire capacity; and for any $w \in (w_1, r)$, $\underline{\underline{\beta}}(w) < \beta < \underline{\beta}(w)$, implying that cheap talk will be used to exchange information.

Lemma E6. $\underline{\beta}(w)$ increases in *w*, where $\lim_{w\to c^+} \underline{\beta}(w) = 0$ and $\lim_{w\to r^-} \underline{\beta}(w) = 1$.

Proof. Following the proof of Theorem 1, if $\underline{\beta} > 0$, it satisfies

$$z_r(\underline{\beta})\left(F^{-1}(z_r(\underline{\beta})) - F^{-1}(z_s)\right) + \left(z_r + \underline{\beta}(1-z_r)\right)\int_{\underline{\varepsilon}}^{F^{-1}(z_s)} F(x)dx - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\underline{\beta}))} F(x)dx = 0. \qquad \text{(E.9)}$$

Substituting $z_r(\underline{\beta}) = \dfrac{r - w + \underline{\beta}(w-c)}{r}$, $z_r = 1 - \dfrac{w}{r}$ and $z_s = 1 - \dfrac{c}{w}$ into (E.9), it becomes

$$\frac{r-w+\underline{\beta}(w-c)}{r}\left(F^{-1}\left(\frac{r-w+\underline{\beta}(w-c)}{r}\right)-F^{-1}\left(1-\frac{c}{w}\right)\right)$$
$$+\left(\frac{r-w+\underline{\beta}w}{r}\right)\int_{\underline{\varepsilon}}^{F^{-1}\left(1-\frac{c}{w}\right)}F(x)dx-\int_{\underline{\varepsilon}}^{F^{-1}\left(\frac{r-w+\underline{\beta}(w-c)}{r}\right)}F(x)dx=0 \qquad \text{(E.10)}$$

By applying implicit differentiation of (E.10) with respect to *w*, we get

$$\left(\frac{w-c}{r}\frac{d\underline{\beta}}{dw}-\frac{1-\underline{\beta}}{r}\right)\left(F^{-1}\left(\frac{r-w+\underline{\beta}(w-c)}{r}\right)-F^{-1}\left(1-\frac{c}{w}\right)\right)$$
$$+\frac{r-w+\underline{\beta}(w-c)}{r}\left(\frac{d}{dw}F^{-1}\left(\frac{r-w+\underline{\beta}(w-c)}{r}\right)-\frac{d}{dw}F^{-1}\left(1-\frac{c}{w}\right)\right)$$
$$+\left(\frac{w}{r}\frac{d\underline{\beta}}{dw}-\frac{1-\underline{\beta}}{r}\right)\int_{\underline{\varepsilon}}^{F^{-1}\left(1-\frac{c}{w}\right)}F(x)dx+\left(\frac{r-w+\underline{\beta}w}{r}\right)\left(1-\frac{c}{w}\right)\frac{d}{dw}F^{-1}\left(1-\frac{c}{w}\right) \qquad \text{(E.11)}$$
$$-\frac{r-w+\underline{\beta}(w-c)}{r}\frac{d}{dw}F^{-1}\left(\frac{r-w+\underline{\beta}(w-c)}{r}\right)=0$$

Let $\Psi(w)\equiv F^{-1}\left(\frac{r-w+\underline{\beta}(w-c)}{r}\right)-F^{-1}\left(1-\frac{c}{w}\right)+\int_{\underline{\varepsilon}}^{F^{-1}\left(1-\frac{c}{w}\right)}F(x)dx$. Then, from (E.11) we extract:

$$\frac{d\underline{\beta}}{dw}=\frac{(1-\underline{\beta})\Psi(w)+\dfrac{c^2(r-w)}{w^3 f\left(F^{-1}(1-c/w)\right)}}{(w-c)\Psi(w)+c\displaystyle\int_{\underline{\varepsilon}}^{F^{-1}\left(1-\frac{c}{w}\right)}F(x)dx} \qquad \text{(E.12)}$$

Since, by the definition of $\underline{\beta}$, $z_r(\underline{\beta})\geq z_s$ (otherwise the retailer would have been satisfied with the capacity the supplier builds for free), and since $\Psi(w)=F^{-1}\left(z_r(\underline{\beta})\right)-F^{-1}(z_s)+\int_{\underline{\varepsilon}}^{F^{-1}(z_s)}F(x)dx$, then $\Psi(w)\geq 0$ for any $w\in(c,r)$, implying that $\frac{d\underline{\beta}}{dw}>0$ for any $w\in(c,r)$.

Suppose $w\to c^+$. Then, by the proof of Lemma E1, $\lim_{w\to c^+}\underline{\beta}(w)=0$ because

$$\lim_{w\to c^+} G(0) = \lim_{w\to c^+}\left[(1-w/r)\left(F^{-1}(1-w/r)-F^{-1}(1-c/w)+\int_{\underline{\varepsilon}}^{F^{-1}(1-c/w)} F(x)dx\right)-\int_{\underline{\varepsilon}}^{F^{-1}(1-w/r)} F(x)dx\right]$$

$$= (1-c/r)\left(F^{-1}(1-c/r)-\underline{\varepsilon}\right)-\int_{\underline{\varepsilon}}^{F^{-1}(1-c/r)} F(x)dx$$

$$\geq (1-c/r)\left(F^{-1}(1-c/r)-\underline{\varepsilon}\right)-\int_{\underline{\varepsilon}}^{F^{-1}(1-c/r)} F(F^{-1}(1-c/r))dx = 0$$

Suppose $w \to r^-$. Then, by the proof of Lemma E1, $\lim_{w\to r^-}\underline{\beta}(w)=1$ because

$$\lim_{w\to r^-} G(1) = \lim_{w\to r^-}\left[(1-c/r)\left(F^{-1}(1-c/r)-F^{-1}(1-c/w)\right)-\int_{F^{-1}(1-c/w)}^{F^{-1}(1-c/r)} F(x)dx\right]$$

$$\leq \lim_{w\to r^-}\left[(1-c/r)\left(F^{-1}(1-c/r)-F^{-1}(1-c/w)\right)-\int_{F^{-1}(1-c/w)}^{F^{-1}(1-c/r)} F(F^{-1}(1-c/w))dx\right],$$

$$= \lim_{w\to r^-}\left[(1-w/r)\left(F^{-1}(1-c/r)-F^{-1}(1-c/w)\right)\right] = 0^+$$

and because $z_r(\beta)\geq z_s\ (\Leftrightarrow \beta \geq \dfrac{w-cr/w}{w-c})$ implies that β approaches 1^- when w approaches r^-. Hence, the lemma is proved. ■

Lemma E7. When $\underline{\underline{\beta}}<1$, $\underline{\underline{\beta}}(w)$ decreases in *w*, where $\lim_{w\to r^-}\underline{\underline{\beta}}(w)=0$.

Proof. By substituting $z_r = 1-\dfrac{w}{r}$ and $z_s = 1-\dfrac{c}{w}$ into Theorem 2(i), we get

$$\underline{\underline{\beta}} = \left(\frac{r}{w}-1\right)\left(\frac{c}{c-w(1-R(\Delta))}-1\right). \tag{E.13}$$

In the proof, we use the following two properties of $R(\Delta)$:

$$1-\frac{c}{w}=z_s=\frac{1}{\Delta}\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(F^{-1}(z_s))dx < R(\Delta)=\frac{1}{\Delta}\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} F(x)dx < \frac{1}{\Delta}\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)} dx = 1$$

$$\frac{dR(\Delta)}{dw}=\frac{c}{w^2}\,\frac{F\left(\Delta+F^{-1}\left(1-\frac{c}{w}\right)\right)-\left(1-\frac{c}{w}\right)}{\Delta\cdot f\left(F^{-1}\left(1-\frac{c}{w}\right)\right)}>0$$

By differentiating $\underline{\underline{\beta}}$ with respect to *w*, we obtain

$$\frac{d\underline{\underline{\beta}}}{dw} = -\frac{r}{w^2}\left(\frac{c}{c-w(1-R(\Delta))}-1\right)+\left(\frac{r}{w}-1\right)\frac{c}{(c-w(1-R(\Delta)))^2}\left(1-R(\Delta)-w\frac{dR(\Delta)}{dw}\right)$$
$$= \frac{-r(1-R(\Delta))(c-w(1-R(\Delta)))+c(r-w)(1-R(\Delta)-wR'(\Delta))}{w(c-w(1-R(\Delta)))^2}$$
$$\leq \frac{-r(1-R(\Delta))(c-w(1-R(\Delta)))+c(r-w)(1-R(\Delta))}{w(c-w(1-R(\Delta)))^2} = -\frac{r(1-R(\Delta))\left[R(\Delta)-(1-c/r)\right]}{(c-w(1-R(\Delta)))^2}.$$

Hence, $\frac{d\underline{\underline{\beta}}}{dw} < 0$ if and only if $R(\Delta) > 1-c/r$. By extracting $R(\Delta)$ from the formula in (E.13), we get $R(\Delta) = 1-\frac{\underline{\underline{\beta}}c}{r-w+\underline{\underline{\beta}}w}$, so the condition $R(\Delta) > 1-c/r$ is simplified to $\underline{\underline{\beta}} < 1$. This means that when $\underline{\underline{\beta}} < 1$, $\underline{\underline{\beta}}$ decreases in *w*. To complete the proof, we use the relation $1-\frac{c}{w} < R(\Delta)$ for any *w*, and calculate

$$\lim_{w\to r^-}\underline{\underline{\beta}}(w) = \lim_{w\to r^-}\left(\frac{r}{w}-1\right)\left(\frac{c}{c-w(1-R(\Delta))}-1\right) = 0.$$

Hence, the lemma is proved. ■

By Lemma E7, $\underline{\underline{\beta}}(w)$ either decreases monotonically over the domain (c,r), or there exists $w_3 \in (c,r)$ such that, for $w \in (c,w_3)$, $\underline{\underline{\beta}}(w) > 1$, and for $w \in (w_3,r)$, $\underline{\underline{\beta}}(w) < 1$, $\underline{\underline{\beta}}(w)$ decreases monotonically, and it approaches 0 when *w* approaches *r*.

By combining the results of Lemma E6 and Lemma E7, the claim of Proposition 5 is proved.

Proof of Proposition 6

From the supplier's perspective:

(i) When $\beta \geq \underline{\beta}$, the retailer finances the entire capacity (i.e., $q^{adv} = \mu_i + F^{-1}(z_r(\beta))$). In this case, the supplier's ex-ante payoff is given by $\Pi_s = (w-c)\left[p\mu_H + (1-p)\mu_L + F^{-1}(z_r(\beta))\right]$. Since $z_r(\beta)$ increases in β, it is clear that Π_s also increases in β.

(ii) If $\underline{\underline{\beta}} \leq \beta < \underline{\beta}$, information is exchanged via cheap talk. Therefore, the supplier's ex-ante payoff is given by $\Pi_s = p\pi_s\left(\mu_H + F^{-1}(z_S) \mid \mu_H, 0\right) + (1-p)\pi_s\left(\mu_L + F^{-1}(z_S) \mid \mu_L, 0\right)$, which is indeed independent of β.

(iii) When $\beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\}$ and the signaling game is played, the retailer orders zero units in advance under the low-demand case and $\tilde{q} < \mu_H + F^{-1}(z_s)$ under the high-demand case. Therefore, the supplier's ex-ante payoff is $\Pi_s = p\pi_s\left(\mu_H + F^{-1}(z_S) \mid \mu_H, \tilde{q}\right) + (1-p)\pi_s\left(\mu_L + F^{-1}(z_S) \mid \mu_L, 0\right)$. Note that $\pi_s\left(\mu_L + F^{-1}(z_S) \mid \mu_L, 0\right)$ is independent of β and $\pi_s\left(\mu_H + F^{-1}(z_S) \mid \mu_H, \tilde{q}\right)$ increases in $\tilde{q}$. Recall that if $\underline{\underline{\beta}} \geq 1$, the separating quantity $\tilde{q}$ increases in β, implying that Π_s also increases in β. However, if $\underline{\underline{\beta}} < 1$, the separating quantity $\tilde{q}$ decreases in β, implying that Π_s also decreases in β.

(iv) When $\beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\}$ and the pooling game is played, the retailer orders zero units in advance under both the low- and the high-market state. Therefore, the supplier builds a capacity K^P, which is independent of β, implying that Π_s is also independent of β.

From the retailer's perspective:

Based on the optimal decisions on the equilibrium path, and when the separating equilibrium is played, the retailer anticipates that

$$\Pi_r = \begin{cases} p\pi_r\left(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H\right) + (1-p)\pi_r\left(0, \mu_L + F^{-1}(z_s) \mid \mu_L\right) & \beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\} \\ p\pi_r\left(0, \mu_H + F^{-1}(z_s) \mid \mu_H\right) + (1-p)\pi_r\left(0, \mu_L + F^{-1}(z_s) \mid \mu_L\right) & \beta \in [\underline{\underline{\beta}}, \underline{\beta}] \\ p\pi_r\left(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H\right) + & \\ \quad + (1-p)\pi_r\left(\mu_L + F^{-1}(z_r(\beta)), \mu_L + F^{-1}(z_r(\beta)) \mid \mu_L\right) & \beta > \underline{\beta} \end{cases}$$

When $\underline{\underline{\beta}} \geq \underline{\beta}$, the middle possibility vanishes, and only the first and last possibilities are considered.

(a) When $\beta < \min\{\underline{\underline{\beta}}, \underline{\beta}\}$, a signaling game exists between the retailer and the supplier. We note that $\pi_r\left(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H\right)$ is a decreasing function of $\tilde{q}$. By Proposition 2, when $\underline{\underline{\beta}} \geq 1$, $\tilde{q}$ increases in β, implying that $\pi_r\left(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H\right)$ decreases in β; however, when $\underline{\underline{\beta}} < 1$, $\tilde{q}$ decreases in β.

(b) When $\underline{\underline{\beta}} \leq \beta < \underline{\beta}$, a cheap-talk equilibrium exists and Π_r is independent of β.

(c) When $\beta \geq \underline{\beta}$, the retailer finances the entire capacity, where its optimal value is determined so as to maximize v_r. We next show that in this region, the function Π_r decreases in β.

Lemma E8. $\pi_r\left(\mu_i + F^{-1}(z_r(\beta))\,,\,\mu_i + F^{-1}(z_r(\beta))\,|\,\mu_i\right)$ decreases in β.

Proof. By equation (3),

$$\pi_r\left(\mu_i + F^{-1}(z_r(\beta))\,,\,\mu_i + F^{-1}(z_r(\beta))\,|\,\mu_i\right) = r\left[z_r\left(\mu_i + F^{-1}(z_r(\beta))\right) - \int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx\right].$$

Using the relations $\dfrac{dz_r(\beta)}{d\beta} = z_s(1-z_r)$ and $\dfrac{d}{d\beta}\displaystyle\int_{\underline{\varepsilon}}^{F^{-1}(z_r(\beta))} F(x)dx = z_r(\beta)\dfrac{dF^{-1}(z_r(\beta))}{d\beta}$, we get

$$\frac{d\pi_r\left(\mu_i + F^{-1}(z_r(\beta))\,,\,\mu_i + F^{-1}(z_r(\beta))\,|\,\mu_i\right)}{d\beta} = r\left(z_r - z_r(\beta)\right)\frac{dF^{-1}(z_r(\beta))}{d\beta}$$

$$= -r\beta z_s(1-z_r)\frac{1}{f\left(F^{-1}(z_r(\beta))\right)}\frac{d\left(z_r(\beta)\right)}{d\beta} = -\frac{r\beta z_s{}^2(1-z_r)^2}{f\left(F^{-1}(z_r(\beta))\right)} < 0. \blacksquare$$

By the definition of Π_r and Lemma E8, we conclude that Π_r decreases in β.

Finally, note that when the pooling equilibrium is played, both retailer types order zero units, and the retailer determines a capacity that is independent of the financial holdings level. Therefore, the ex-ante operational payoff of the retailer, when the pooling equilibrium is played, is independent of β.

Proofs of claims in Appendix B

Proof of Lemma B1

When $\beta = 0$, the supplier's problem is formulated in the following way:

$$\max_{\{(K_H,T_H),(K_L,T_L)\}}\left\{p(T_H - cK_H) + (1-p)(T_L - cK_L)\right\}$$

$$\begin{aligned}
S.t.\quad & rE\left[\min(D,K_H)\,|\,\mu_H\right] - T_H \geq rE\left[\min(D,K_L)\,|\,\mu_H\right] - T_L && (IC_HL)\\
& rE\left[\min(D,K_H)\,|\,\mu_H\right] - T_H \geq 0 && (PC_H)\\
& rE\left[\min(D,K_L)\,|\,\mu_L\right] - T_L \geq rE\left[\min(D,K_H)\,|\,\mu_L\right] - T_H && (IC_LH)\\
& rE\left[\min(D,K_L)\,|\,\mu_L\right] - T_L \geq 0 && (PC_L)
\end{aligned}$$

The first constraint states that the high-type retailer does not wish to mimic the low-type retailer by accepting her contract (incentive compatibility). The second constraint ensures that the high-type retailer prefers to accept her contract, since it affords her a non-negative payoff (participation constraint). In a similar manner, the third constraint ensures that the low-type retailer chooses her contract over the one designed for the high-type retailer (incentive compatibility), and the last constraint ensures that the low-type retailer does not decline her contract (participation constraint).

In this problem, the two binding constraints are the incentive compatibility constraint of the high type (IC_HL) and the participation constraint of the low type (PC_L). Consequently, it is possible to re-frame the supplier's problem in the following manner:

$$\max_{\{K_H,K_L\}} \left\{ \begin{array}{l} p\left[r\left\{ E\left[\min(D,K_H) \mid \mu_H\right] - \left(E\left[\min(D,K_L) \mid \mu_H\right] - E\left[\min(D,K_L) \mid \mu_L\right] \right) \right\} - cK_H \right] \\ +(1-p)\left(rE\left[\min(D,K_L) \mid \mu_L\right] - cK_L \right) \end{array} \right\}.$$

This problem is separable in K_H and K_L, and the solution is outlined in the lemma.

Proof of Proposition B1

(i) The two binding constraints are the high type's incentive compatibility constraint and the low type's participation constraint. Since PC_L is binding, we obtain $T_L = \frac{rE\left[\min(D,K_L) \mid \mu_L\right] - cK_L}{1-\beta} + cK_L$.

Substituting this expression into the incentive compatibility constraint of the high-type retailer, we extract

$$T_H = \frac{r\left[E\left[\min(D,K_H) \mid \mu_H\right] - \left(E\left[\min(D,K_L) \mid \mu_H\right] - E\left[\min(D,K_L) \mid \mu_L\right] \right) \right] - cK_H}{1-\beta} + cK_H .$$

Consequently, the supplier's problem can be formulated as:

$$\max_{\{K_H,K_L\}} \frac{\left\{ \begin{array}{l} p\left[r\left\{ E\left[\min(D,K_H) \mid \mu_H\right] - \left(E\left[\min(D,K_L) \mid \mu_H\right] - E\left[\min(D,K_L) \mid \mu_L\right] \right) \right\} - cK_H \right] \\ +(1-p)\left(rE\left[\min(D,K_L) \mid \mu_L\right] - cK_L \right) \end{array} \right\}}{1-\beta}$$

This problem is separable in K_H and K_L , and the optimal values of these capacities do not depend on the value of β . Therefore, the optimal capacities K_H and K_L are the same for any value of β.

(ii) We differentiate T_L with respect to β and obtain $\frac{dT_L}{d\beta} = \frac{rE\left[\min(D,K_L) \mid \mu_L\right] - cK_L}{(1-\beta)^2} > 0$, since the supply chain is profitable. Since the supplier's payoff is positive, in a similar manner, it is possible to show that

$$\frac{dT_H}{d\beta} = \frac{r\left[E\left[\min(D,K_H) \mid \mu_H\right] - \left(E\left[\min(D,K_L) \mid \mu_H\right] - E\left[\min(D,K_L) \mid \mu_L\right] \right) \right] - cK_H}{(1-\beta)^2} > 0 .$$

Proofs of claims in Appendix C

Proof of Proposition C1

We start by showing that there is no separating equilibrium in which the low-type retailer sells shares and the high type does not. We then prove the converse – that there is no separating equilibrium in which the high type sells shares and the low type does not.

First, assume that the low-type retailer sells her shares and the high type does not. In this case, the decision to sell shares signals to the market and to the supplier that the demand state is low. Therefore, the market prices these shares based on this inference, and the supplier sets a capacity $\mu_L + F^{-1}(z_s)$. Accordingly, the value of the shares is $\beta\pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0)$ and the operational payoff of the low-type retailer is $\pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L)$. By refraining from selling shares, the low type mimics the high type and can achieve the accounting payoff of $\pi_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L) + \beta\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_L, 0)$, which is higher than the payoff she receives from selling shares (in the case of $\beta < \underline{\underline{\beta}}$).

Second, assume that the high type sells shares and the low type does not. Upon offering the shares for sale, the market and the supplier infer that demand is high, and thus the value of the shares is priced at $\beta\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_H, 0)$. If the low-type retailer does not sell shares, she will receive the accounting payoff of $\pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L) + \beta\pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0)$, while by selling shares, she will first receive the payoff of $\beta\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_H, 0)$ for the shares, followed by the operational payoff of $\pi_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L)$. Note that the following inequality holds for any value of β:

$$\pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L) + \beta\pi_s(\mu_L + F^{-1}(z_s) \mid \mu_L, 0) < \pi_r(0, \mu_H + F^{-1}(z_s) \mid \mu_L) + \beta\pi_s(\mu_H + F^{-1}(z_s) \mid \mu_H, 0),$$

such that the low type will always have an incentive to mimic the high type.

Proof of Proposition C2

Assume, to the contrary, that there is a pooling equilibrium in which both retailer types sell the same number of shares in the supplier. Accordingly, no information is revealed based on the act of selling the shares. Two options exist after the sale: in the first, there is an advance order that again fails to reveal any information (i.e., a pooling equilibrium is played also at this stage), while in the second option, there is information revelation based on the advance order.

First, assume that the advance order does not reveal any information (i.e., a pooling equilibrium with zero units ordered in advance). In this scenario, the value of the sold shares is $\beta\left[p\pi_s(K^P \mid 0, \mu_H) + (1-p)\pi_s(K^P \mid 0, \mu_L)\right]$, and the operational payoff of the high-type retailer is $\pi_r(0, K^P \mid \mu_H)$. If, on the other hand, the retailer had not sold any shares (meaning that the low-type retailer mimicked the high-type retailer), then the high-type retailer earns $\pi_r(0, K^P \mid \mu_H) + \beta\pi_s(K^P \mid 0, \mu_H)$, which is higher than her total payoff when selling the shares.

In the second option, after selling the shares, the retailer signals the state of the demand to the supplier based on the advance-order quantity. In this case, the shares are sold for the value

$\beta\left[p\pi_s(\mu_H + F^{-1}(z_s) \mid \tilde{q}, \mu_H) + (1-p)\pi_s(\mu_L + F^{-1}(z_s) \mid 0, \mu_H) \right]$ based on the anticipation of playing a separating equilibrium. However, if the retailer had opted not to sell any shares, then she would have realized an accounting payoff of $\beta\pi_s(\mu_H + F^{-1}(z_s) \mid \tilde{q}, \mu_H)$, which is higher than the value of the shares when sold based on prior information only. Therefore, the retailer chooses to keep her shares.

Proof of Proposition C3

The retailer solves $\max_{\beta}\{\Pi_r\}$ based on the subsequent equilibrium played for any value of β.

(i) First, assume that the pooling equilibrium is played on the equilibrium path. In this case, the ex-ante operational payoff of the retailer is $p\pi_r(0, K^P \mid \mu_H) + (1-p)\pi_r(0, K^P \mid \mu_L)$, which does not depend on β. Therefore, we conclude that, in this case, no shares will be purchased.

(ii) Second, assume that the separating equilibrium is played on the equilibrium path. In this case, the ex-ante operational payoff of the retailer is given by

$$p\pi_r(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H) + (1-p)\pi_r(0, \mu_L + F^{-1}(z_s) \mid \mu_L) .$$

Note that the second element does not depend on β, while the first element, through its dependence on the separating quantity $\tilde{q}$, in turn depends on β (see Proposition 2). If $\tilde{q}$ increases in β, then $\pi_r(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H)$ decreases in β because it decreases in $\tilde{q}$. In this case, the retailer chooses not to purchase any shares. However, if $\tilde{q}$ decreases in β, then $\pi_r(\tilde{q}, \mu_H + F^{-1}(z_s) \mid \mu_H)$ increases in β, implying that the retailer wishes to purchase the highest possible number of shares in the supplier so as to minimize the signaling cost. Since, when $\underline{\underline{\beta}} \le \beta < \underline{\beta}$, a cheap-talk equilibrium is played, and since, when $\beta \ge \underline{\beta}$, the retailer finances the entire capacity in the market, the retailer chooses $\beta^* = \min\left\{\underline{\beta}, \underline{\underline{\beta}}\right\}$.

(iii) When $\underline{\underline{\beta}} \le \beta < \underline{\beta}$, information is exchanged via cheap talk. In this region, the ex-ante operational payoff of the retailer is independent of β, and therefore the retailer purchases the lowest possible share in the supplier, which in this case is $\underline{\underline{\beta}}$.

(iv) The last case to consider is $\beta \ge \underline{\beta}$. In this scenario, on the equilibrium path, the retailer finances the entire capacity in the market. Her ex-ante operational payoff is given by

$$p\pi_r(\mu_H + F^{-1}(z_r(\beta)), \mu_H + F^{-1}(z_r(\beta)) \mid \mu_H) + (1-p)\pi_r(\mu_{LH} + F^{-1}(z_r(\beta)), \mu_L + F^{-1}(z_r(\beta)) \mid \mu_L) .$$

By Lemma E8, $\pi_r(\mu_i + F^{-1}(z_r(\beta)), \mu_i + F^{-1}(z_r(\beta)) \mid \mu_i), i \in \{L, H\}$, decreases in β. Therefore, in this region, the retailer should purchase a share of $\underline{\beta}$ in the supplier.

Proofs of claims in Appendix D

Proof of Proposition D1

Implicit differentiation of (17) with respect to Δ (for a given μ_L) yields

$$\frac{d\tilde{q}}{d\Delta}=\frac{z_r(1-z_s)-\left(z_r+\beta(1-z_r)\right)\left(R(\Delta)+\Delta\frac{dR(\Delta)}{d\mu_H}-z_s\right)}{(1-\beta)(1-z_r)F(\tilde{q}-\mu_L)}. \tag{E.14}$$

Substituting $\frac{dR(\Delta)}{d\Delta}=\frac{\Delta\cdot F(\Delta+F^{-1}(z_s))-\int_{F^{-1}(z_s)}^{\Delta+F^{-1}(z_s)}F(x)dx}{\Delta^2}=\frac{F(\Delta+F^{-1}(z_s))-R(\Delta)}{\Delta}$ into (E.14) yields

$$\frac{d\tilde{q}}{d\Delta}=\frac{n(\Delta)}{(1-\beta)(1-z_r)F(\tilde{q}-\mu_L)}, \tag{E.15}$$

where $n(\Delta)\equiv z_r(1-z_s)-\left(z_r+\beta(1-z_r)\right)\left(F(\Delta+F^{-1}(z_s))-z_s\right)$.

(i) If $\beta=0$, $\frac{d\tilde{q}}{d\Delta}=\frac{z_r\left(1-F(\Delta+F^{-1}(z_s))\right)}{(1-z_r)F(\tilde{q}-\mu_L)}\geq 0$, which proves the claim.

(ii) If $\beta>0$, the sign of $\frac{d\tilde{q}}{d\Delta}$ is dictated by $n(\Delta)$, where $n(\Delta)$ is a (weakly) decreasing function of Δ (because $F(\cdot)$ is a (weakly) increasing function of its argument), $n(0)=z_r\left(1-F(\Delta+F^{-1}(z_s))\right)\geq 0$ and $\lim_{\Delta\to\infty}n(\Delta)=-\beta(1-z_r)(1-z_s)<0$. Thus, a unique value of Δ,

$$\underline{\Delta}\equiv F^{-1}\left(z_s+\frac{1-z_s}{1+\beta(1/z_r-1)}\right)-F^{-1}(z_s)=F^{-1}\left(R\left(\underline{\underline{\Delta}}\right)\right)-F^{-1}(z_s)>0, \tag{E.16}$$

which satisfies $n\left(\underline{\Delta}\right)=0$ exists, such that $\tilde{q}$ is a quasi-concave function of Δ with a maximum at $\Delta=\underline{\Delta}$.

Proof of Proposition D2

We need to examine the service level type II under the four possible outcomes: when a signaling game exists, when a cheap-talk equilibrium exists, when the retailer finances the entire capacity in the market, and when a pooling equilibrium exists.

Case I – Signaling game. In this case, which arises when $\beta<\min\{\underline{\beta},\underline{\underline{\beta}}\}$, the high-type retailer orders in advance some quantity $\tilde{q}$, and the supplier secures the capacity level $K_H=\mu_H+F^{-1}(z_s)>\tilde{q}$, whereas the low-type retailer orders zero units in advance, and the supplier secures the capacity $K_L=\mu_L+F^{-1}(z_s)$. The service level type II is calculated by

$$SL_{II} = p\left(1 - \frac{1}{\mu_H}\int_{F^{-1}(z_s)}^{\bar{\varepsilon}} (\varepsilon - F^{-1}(z_s))\, f(\varepsilon)d\varepsilon\right) + (1-p)\left(1 - \frac{1}{\mu_L}\int_{F^{-1}(z_s)}^{\bar{\varepsilon}} (\varepsilon - F^{-1}(z_s))\, f(\varepsilon)d\varepsilon\right). \quad \text{(E.17)}$$

It is easy to see that this function is independent of β.

Case II – Cheap-talk equilibrium. In this case, which arises when $\beta \in [\underline{\underline{\beta}}, \underline{\beta})$, the retailer verbally communicates the true market condition to the supplier. The supplier secures the capacity $K_H = \mu_H + F^{-1}(z_s)$ in the high-market state and $K_L = \mu_L + F^{-1}(z_s)$ in the low-market state. Therefore, the service level type II is the same as in Equation (E.17) and is independent of β.

Case III – The retailer finances the capacity. In this case, which arises when $\beta \geq \underline{\beta}$, the retailer orders in advance the quantity $q_i^{adv} = \mu_i + F^{-1}(z_r(\beta))$, $i \in \{L, H\}$, and the supplier builds a capacity to match this order; i.e., $K_i = q_i^{adv}$. Therefore, the service level type II is calculated by

$$SL_{II} = p\left(1 - \frac{1}{\mu_H}\int_{F^{-1}(z_r(\beta))}^{\bar{\varepsilon}} (\varepsilon - F^{-1}(z_r(\beta)))\, f(\varepsilon)d\varepsilon\right) + (1-p)\left(1 - \frac{1}{\mu_L}\int_{F^{-1}(z_r(\beta))}^{\bar{\varepsilon}} (\varepsilon - F^{-1}(z_r(\beta)))\, f(\varepsilon)d\varepsilon\right).$$

Since $z_r(\beta)$ increases in β, the service level type II also increases in β.

Case IV – The pooling equilibrium. In this case, which arises when $\beta \leq \beta^P \in [0, \underline{\beta})$, the retailer orders zero units in advance. Therefore, the supplier builds a capacity $K_L = K_H = K^P$, which is independent of β, implying that the service level type II is also independent of β.

References

Alan, Y. and Gaur, V., 2018. Operational investment and capital structure under asset-based lending. *Manufacturing & Service Operations Management*, 20(4), pp.637-654.

Allen, J.W. and Phillips, G.M., 2000. Corporate equity ownership, strategic alliances, and product market relationships. *The Journal of Finance*, 55(6), pp.2791-2815.

Anand, K.S. and Goyal, M., 2009. Strategic information management under leakage in a supply chain. *Management Science*, 55(3), pp.438-452.

Avinadav, T., Chernonog, T., Fruchter, G. E. and Prasad., A., 2020. Contract Design when Quality is Co-Created in a Supply Chain. *European Journal of Operational Research*, 286, pp.908-918.

Avinadav, T., Chernonog T. and Khmelnitsky, E., 2021. Revenue-sharing between developers of virtual products and platform distributors. *European Journal of Operational Research*, 290, pp.927-945.

Avinadav, T. and Shamir, N., 2021. The effect of information asymmetry on ordering and capacity decisions in supply chains. *European Journal of Operational Research*, 292(2), pp.562-578.

Aviv, Y. and Shamir, N., 2021. Financial cross-ownership and information dissemination in a supply-chain. *Manufacturing & Service Operations Management*, 23(6), pp.1524-1538.

Azar, J. 2012. A new look at oligopoly: Implicit collusion through portfolio diversification. PhD Thesis, Princeton Univ.

Aziz, F.A. and Tolouei, S., 2012. Small block of supplier shares improve trust performance in supply chain relationship. In Pathways to Supply Chain Excellence.

Bajari, P. and Tadelis, S., 2001. Incentives versus transaction costs: A theory of procurement contracts. *The RAND Journal of Economics*, 32(3), pp.387-407.

Babich, V. and Sobel, M.J., 2004. Pre-IPO operational and financial decisions. *Management Science*, 50(7), pp.935-948

Bhaskaran, S., Erzurumlu, S.S. and Ramachandran, K., 2020. Sequential product development and introduction by cash-constrained start-ups. *Manufacturing & Service Operations Management*, forthcoming.

Berman, O., Fazel-Zarandi, M.M. and Krass, D., 2019. Truthful cheap talk: Why operational flexibility may lead to truthful communication. *Management Science*, 65(4), pp.1624-1641.

Cachon, G.P., 2003. Supply chain coordination with contracts. Handbooks in Operations Research and *Management Science*, 11, pp.227-339.

Cachon, G.P. and Lariviere, M.A., 2001. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Science*, 47(5), pp.629-646.

Chen, F., 2003. Information sharing and supply chain coordination. *Handbooks in Operations Research and Management Science*, 11, pp.341-421.

Chen, J., Hu, Q. and Song, J.S., 2017. Effect of partial cross ownership on supply chain performance. *European Journal of Operational Research*, 258(2), pp.525-536.

Cho, I.K. and Kreps, D.M., 1987. Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2), pp.179-221.

Chod, J., 2017. Inventory, risk shifting, and trade credit. *Management Science*, 63(10), pp.3207-3225.

Chod, J. and Zhou, J., 2014. Resource flexibility and capital structure. *Management Science*, 60(3), pp.708-729

Chu, L.Y., Shamir, N. and Shin, H., 2017. Strategic communication for capacity alignment with pricing in a supply chain. *Management Science*, 63(12), pp.4366-4388.

Cohen, M.A., Ho, T.H., Ren, Z.J. and Terwiesch, C., 2003. Measuring imputed cost in the semiconductor equipment supply chain. *Management Science*, 49(12), pp.1653-1670.

Crawford, V.P. and Sobel, J., 1982. Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pp.1431-1451.

Desai, P.S. and Srinivasan, K., 1995. Demand signalling under unobservable effort in franchising: Linear and nonlinear price contracts. *Management Science*, 41(10), pp.1608-1623.

Drees, F., Mietzner, M. and Schiereck, D., 2013. Effects of corporate equity ownership on firm value. *Review of Managerial Science*, 7(3), pp.277-308.

Dyer, J.H. and Ouchi, W.G., 1993. Japanese-style partnerships: giving companies a competitive edge. *MIT Sloan Management Review*, 35(1), p.51-63.

Dyer, J.H. and Hatch, N.W., 2004. Using supplier networks to learn faster. *MIT Sloan Management Review*, 45(3), p.57.

Fang, F., Jiang, B. and Sun, J., 2021. Block ownership in vertical relationships in the presence of downstream competition. Working paper, http://dx.doi.org/10.2139/ssrn.3501303

Fee, C.E., Hadlock, C.J. and Thomas, S., 2006. Corporate equity ownership and the governance of product market relationships. *The Journal of Finance*, 61(3), pp.1217-1251.

Gibbons, R.S., 1992. Game theory for applied economists. Princeton University Press.

Gilo, D. and Spiegel, Y., 2011. Partial vertical integration in telecommunication and media markets in Israel. *Israel Economic Review*, 9(1).

Gilo, D., Moshe, Y. and Spiegel, Y., 2006. Partial cross ownership and tacit collusion. *The RAND Journal of Economics*, 37(1), pp.81-99.

Greenlee, P. and Raskovich, A., 2006. Partial vertical ownership. *European Economic Review*, 50(4), pp.1017-1041.

Ha, A.Y., 2001. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Research Logistics*, 48(1), pp.41-64.

Ha, A.Y. and Tang, C.S., 2017. *Handbook of Information Exchange in Supply Chain Management*. Springer.

Ha, A.Y. and Tong, S., 2008. Contracting and information sharing under supply chain competition. *Management Science*, 54(4), pp.701-715.

He, J.J. and Huang, J., 2017. Product market competition in a world of cross-ownership: Evidence from institutional blockholdings. *The Review of Financial Studies*, 30(8), pp.2674-2718.

Iancu, D.A., Trichakis, N. and Tsoukalas, G., 2017. Is operating flexibility harmful under debt?. *Management Science*, 63(6), pp.1730-1761

Iyer, G. and Villas-Boas, J.M., 2003. A bargaining theory of distribution channels. *Journal of Marketing Research*, 40(1), pp.80-100.

Jiang, B., Tian, L., Xu, Y. and Zhang, F., 2016. To share or not to share: Demand forecast sharing in a distribution channel. *Marketing Science*, 35(5), pp.800-809.

Kong, G., Rajagopalan, S. and Zhang, H., 2013. Revenue sharing and information leakage in a supply chain. *Management Science*, 59(3), pp.556-572.

Kwon, I.W.G. and Suh, T., 2004. Factors affecting the level of trust and commitment in supply chain relationships. *Journal of supply chain management*, 40(1), pp.4-14.

Law Right. 2016. The separation between ownership and control in companies: A key to success. Available at https://www.law-right.com/the-separation-between-ownership-and-control-in-companies/

Lai, G., Debo, L. and Nan, L., 2011. Channel stuffing with short-term interest in market value. *Management Science*, 57(2), pp.332-346.

Lai, G. and Xiao, W., 2018. Inventory decisions and signals of demand uncertainty to investors. *Manufacturing & Service Operations Management*, 20(1), pp.113-129.

Lai, G., Xiao, W. and Yang, J., 2012. Supply chain performance under market valuation: An operational approach to restore efficiency. *Management Science*, 58(10), pp.1933-1951.

Lee, H.L., So, K.C. and Tang, C.S., 2000. The value of information sharing in a two-level supply chain. *Management Science*, 46(5), pp.626-643.

Levy, N., Spiegel, Y. and Gilo, D., 2018. Partial vertical integration, ownership structure, and foreclosure. *American Economic Journal: Microeconomics*, 10(1), pp.132-180.

Li, L., 2002. Information sharing in a supply chain with horizontal competition. *Management Science*, 48(9), pp.1196-1212.

Li, X., Özer, Ö. and Subramanian, U., 2020. Are we strategically naïve or guided by trust and trustworthiness in cheap-talk communication?. *Management Science*, forthcoming.

Li, L. and Zhang, H., 2002. Supply chain information sharing in a competitive environment. In Supply Chain Structures (pp. 161-206). Springer, Boston, MA.

Li, L. and Zhang, H., 2008. Confidentiality and information sharing in supply chain coordination. *Management Science*, 54(8), pp.1467-1481.

Lu, R., Shen, Q., Wang, T. and Zhang, X., 2021. Frenemies: Corporate Advertising Under Common Ownership. *Management Science*, forthcoming.

Mailath, G.J., Okuno-Fujiwara, M. and Postlewaite, A., 1993. Belief-based refinements in signalling games. *Journal of Economic Theory*, 60(2), pp.241-276.

Magretta, J., 1998. The power of virtual integration: An interview with Dell Computer's Michael Dell. *Harvard Business Review*, 76(2), pp.72-83.

Myerson, R.B., 1979. Incentive compatibility and the bargaining problem. *Econometrica*, pp.61-73.

Özer, Ö., Subramanian, U. and Wang, Y., 2018. Information sharing, advice provision, or delegation: what leads to higher trust and trustworthiness?. *Management Science*, 64(1), pp.474-493.

Özer, Ö. and Wei, W., 2006. Strategic commitments for an optimal capacity decision under asymmetric forecast information. *Management Science*, 52(8), pp.1238-1257.

Özer, Ö., Y. Zheng., 2019. The Handbook of Behavioral Operations, chap. Trust and Trustworthiness. Wiley Series in Operations Research and Management Science, pp.489-524

Özer, Ö., Zheng, Y. and Chen, K.Y., 2011. Trust in forecast information sharing. *Management Science*, 57(6), pp.1111-1137.

Peura, H., Yang, S.A. and Lai, G. 2017. Trade credit in competition: a horizontal benefit. *Manufacturing Service Operations Management*, 19(2), pp.263-289

Reiffen, D., 1998. Partial ownership and foreclosure: an empirical analysis. *Journal of Regulatory Economics*, 13(3), pp.227-244.

Ren, Z.J., Cohen, M.A., Ho, T.H. and Terwiesch, C., 2010. Information sharing in a long-term supply chain relationship: The role of customer review strategy. *Operations Research*, 58(1), pp.81-93.

Schmalz, M.C., 2018. Common-ownership concentration and corporate conduct. *Annual Review of Financial Economics*, 10, pp.413-448.

Schmidt, W., Gaur, V., Lai, R. and Raman, A., 2015. Signaling to partially informed investors in the newsvendor model. *Production and Operations Management*, 24(3), pp.383-401.

Semov, S., 2017. Common ownership, competition and firm financial policy. Boston University Working Paper.

Shamir, N. and Shin, H., 2016. Public forecast information sharing in a market with competing supply chains. *Management Science*, 62(10), pp.2994-3022.

Shamir, N. and Shin, H., 2018. The perils of sharing information in a trade association under a strategic wholesale price. *Production and Operations Management*, 27(11), pp.1978-1995.

Shelegia, S. and Spiegel, Y., 2012. Bertrand competition when firms hold passive ownership stakes in one another. *Economics Letters*, 114(1), pp.136-138.

Spengler, J.J., 1950. Vertical integration and antitrust policy. *Journal of Political Economy*, 58(4), pp.347-352.

Tian, L. and Jiang, B. 2017. Comment on “strategic information management under leakage in a supply chain”. *Management Science*, 63(12), pp.4258-4260.

Tunca, T.I. and Zhu, W. 2018. Buyer intermediation in supplier finance. *Management Science*, 64(12), pp.5631-5650.

Yang, S.A. and Birge, J.R., 2018. Trade credit, risk sharing, and inventory financing portfolios. Management Science, 64(8), pp.3667-3689.

Yoo, O.S., Corbett, C.J., and Roels, G., 2016. Optimal time allocation for process improvement for growth-focused entrepreneurs. *Manufacturing & Service Operations Management*, 18(3), pp.361-375.

Yoo, O.S., Roels, G., and Corbett, C.J., 2016. The time–money trade-off for entrepreneurs: When to hire the first employee? *Manufacturing & Service Operations Management*, 18(4), pp.559-569.