The cost-benefit approach to an optimal charging scheme for an embryo storage service

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Abstract

Cryostorage of human embryos obtained during the course of *in vitro* fertilization treatments is an important issue for hospitals, governments, and individuals who face fertility challenges. Embryo cryostorage is associated with a number of economic, ethical and legal concerns due to rising holding and operational costs, combined with increasing quantities of unused embryos and a lack of economic incentive for hospitals to provide free cryostorage services. These concerns negatively lessen the incentive that providers have to offer at all cryostorage services, while individuals are motivated to seek embryo donations from abroad and thus potentially engage in risky or illegal purchases. As nowadays both public and private healthcare institutions are economically motivated increasing their economic incentive has the potential to positively address the above flaws. This paper proposes a nonlinear programming model for enabling a service provider such as a hospital to determine the optimal prices that it should charge for embryo storage services. The optimal pricing policy is presented analytically and excludes Three-Part Tariffs. Finally, the paper introduces a numerical example as well as a real-data comparison among several providers to show the applicability and highlights the significance of the proposed model.

Keywords: Pricing policy; Revenue management; Decision making; Three-Part Tariff; IVF

1. Introduction

1.1 Motivation

Cryostorage of human embryos has been practiced since the 1980's and is becoming increasingly common due to technological and medical developments in recent years (Michelmann and Nayudu, 2006). Individuals that want to maintain the option of using their embryos in the future in order to expand their families can utilize embryo storage service. Human embryos are usually obtained for cryostorage during the course of *in vitro* fertilization (IVF) treatments. They enable individuals facing fertility challenges to conceive, either using in the future their own embryos or embryos donated by others. Such a service is needed in order to avoid risky and costly new IVF treatments. For these purposes human embryos are kept in a dedicated storage facility. In recent years, the growing scale of embryo storage (Goedeke and Payne, 2009; Kovacs et al., 2003) has raised the following economic, ethical, and legal issues.

(a) Embryo cryostorage service providers that are usually hospitals face rising holding and operational costs. According to a survey conducted in 2012 at the Assuta Hospital in Israel (Lahav, 2013), embryologists estimate that holding and operational costs incurred for a single embryo are approximately $2000 per year. Yet most providers either public or private offer such services for free or at minimal costs. Public and private non-profitable providers (such as hospitals and research institutes) may charge often minimal fees and sometimes even they provide the service for free due to altruistic reasons or due to national aspirations, such as to promote a higher population growth in the country.

(b) Current embryo storage technologies are not fully protected against unexpected damage to the embryos. In such cases, providers are exposed to possible legal proceedings and financial penalties due to the high sensitivity associated with losing embryos. This problem has recently worsened due to an increasing quantity of embryos that are not planned to be used by their owners (Goedeke and Payne, 2009). In Israel, for example, the estimated number of unused embryos is 200,000 (http://www.ynet.co.il/articles/0,7340,L-4928582,00.html).

(c) In most cases donations of unused embryos to a secondary market are made on a voluntary basis (De Lacy, 2007; McMahon et al., 2003; Provoost et al., 2009), thus the demand for donations is not usually met. In spite of existing large amounts of unused embryos, such donations are relatively rare due to regulatory obstacles, ethical difficulties, and low willingness to donate.

(d) Different from regular products keeping remaining unused embryos in storage for practically unlimited time has negative ethical, and legal implications. Discarding unused embryos in inevitable in order to reduce the growing inventory and reducing the possibility of legal proceedings as mentioned above. Of course, in addition to the above complexities, economic considerations such as holding costs of unused inventory and capacity problems are encouraging the process of discarding. Discarding unused embryos without preliminary agreement may expose providers to regulatory complications or legal proceedings.

As nowadays both public and private healthcare institutions are economically motivated, increasing their economic incentive as suggested in our paper, has the potential to positively address the above flaws. Among those economic incentives associated with many hospitals that provide embryo storage services, are limiting the embryo storage period as indicated by (Mohler-Kuo et al., 2009) and reducing regulations to enable those providers to charge higher prices than currently being charged.

1.2 Problem presentation

As a result of the issues raised above, providers have little incentive to offer cryostorage services, while individuals are motivated to seek embryo donations from abroad and thus potentially engage in risky or illegal purchases.

The development of new methods for coping with these challenges is therefore essential for the operation of an embryo cryostorage service. This research assumes a contract of limited duration between the service provider such as a hospital, and individuals requesting the permitted embryo storage service. Limited contracts for storage service are discussed also by Brent and Sunwoong (2003) for modeling the Korean Chonsei lease similarly however in different environment Baker et al. (2006) criticize the long period of digital storage due to reliability difficulties. The suggested pricing scheme addresses the issues described above and creates a financial incentive for the provider to offer embryo cryostorage services, while it also decreases the incentive for individuals to keep unused embryos. Accordingly, the paper proposes a pricing scheme for use by the service provider that charges for embryo storage in the facility during the contract period. The proposed pricing structure maximizes the provider's profits, reduces the quantity of unused embryos in storage, and increases welfare.

1.3 Embryo cryostorage characteristics

The studies in the literature have not directly investigated the problem of pricing embryo storage services. Thus the short survey below is drawn from literature concerning other services that share certain similarities with embryo cryostorage.

(a) Long shelf life: Embryo cryostorage differs in several ways from the storage of other cold chain products, including biological products. Other types of cells can be stored for only limited periods of time. For example, the shelf life of red blood cells has recently been extended to a maximum of 42 days. However, cryopreserved embryos have no expiration date and can be stored for years in liquid nitrogen (at -196 °C) for use in repeated IVF treatments (e.g., Dowling-Lacey et al., 2011; Provoost et al., 2010; Quintans et al., 2002; Almog and Ben-Zeev, 1996; Bar-Hava and Shinkman, 2008). Dowling-Lacey et al. (2011) describe a case in which a healthy baby boy was born from an embryo kept in cryostorage for 20 years.

(b) Negligible storage space: Embryo storage services, similar to data storage services, require negligible storage space and actually have no expiration date. Wang et al. (2010) discuss the use of a "pay as you go" model in cloud computing. Ibrahim et al. (2011) propose another pricing model, "pay as you consume," in which users are charged according to their effective resource consumption.

(c) Limited service duration: An analogy can be made between embryo storage services and services provided by parking lots. The vehicle owner can determine when to end the parking service, subject to the limitation of the parking lot opening hours. Similarly, individuals use a hospital service to store their embryos for a duration limited by the provider, while they remain the only owners of their own embryos.

(d) Risk exposure: With respect to embryo storage services, the probability of damaging the embryo increases with the storage period, and thus financial penalties might also be incurred. Herbon et al. (2014) address managing perishables and claim that when unexpected damage occurs, the retailer loses his reputation and incurs the cost of compensation which is usually significantly higher than the unit cost of the perishable.

1.4 Pricing schemes

Storage service literature provides several common payment methods for a given service. Tsai and Chu (2006) suggest a Stackelberg game between the government (the leader) and the private parking firm (the follower) to address the situation in which a private parking lot must determine fees charged to individual consumers. The leader's objective is welfare maximization, while the private firm's objective is profit maximization.

Another type of pricing structure used to maximize the profit of service providers is the flat rate policy of a fixed price. This type of policy which is used, for example, in "all you can eat" restaurants has been shown to potentially lead to waste since it encourages over consumption. A flat price policy for embryo cryostorage might result in negative outcomes such as the storage of larger quantities of embryos for longer durations.

Included among many examples which overcome the drawbacks of the fixed pricing policy is the Two-Part Tariff literature (Oi, 1971). Courcoubetis and Weber (2003) provide an example of a taxi service in which the customer is charged a as an initial payment for ordering the taxi, while the service price depends on both parameters T and X, that are the duration and the distance of the ride, respectively. The authors show that the price $a + b\cdot T + c\cdot X$ generates an incentive for the taxi driver to provide shorter rides during more congested hours. They also argue that this type of pricing structure can be used to channel the demand according to the congestion level. Narayanan et al. (2007) and Iyenger et al. (2008) describe a similar pricing structure in which the fixed price entry fee includes a limited mileage level, while mileage accrued beyond this level is associated with an additional charge per unit.

Another common strategy is dynamic pricing that assigns an increasing or decreasing price over time. According to Herbon et al. (2014) who analyze a perishable inventory system, price differentiation strategy increases the quantities of products sold and reduces obsolescence. Similarly in the context of the present study, the suggested pricing scheme utilizes dynamic pricing in order to increase the profits of the embryo storage provider and to minimize unused embryos.

1.5 Contribution of the current research

The study draws insights from these previous models and applies them to the case of embryo storage, as further described below.

(a) The suggested model is unique in existing literature as it represents the problem described above by using an operational research approach and by explicitly quantifying the decisions required for implementing the optimal pricing scheme. To the best of our knowledge, existing literature lacks of papers suggesting mathematical modeling of embryo storage service.

(b) Restricting the contract length according to the suggested model contributes to decreasing of remaining unused embryos. Cryopreserved embryos that in many cases remain unused following IVF treatments can be utilized for research purposes. One example is stem cell research aimed at finding cures for degenerative diseases such as Parkinson's syndrome, Alzheimer’s disease, ALS, multiple sclerosis and others. Limitation of the storage contract period enables utilization of unused embryos for such purposes.

(c) We show that the optimal pricing scheme should be an increasing price over time along a finite selling horizon that follows Herbon (2014). Such a pricing scheme is not commonly presented in economic literature.

(d) The implementation of the model can generate Pareto improvement from the point of view of the various “economic agents”:

    (i) Individuals that want to maintain the option of using the service of storage embryos most likely may find more providers;

    (ii) The storage service supplier which is the hospital that provides the storage also benefits financially;

    (iii) The social planner that enables this storage service monitors the process discarding unused embryos and maintains the option of embryo donation either for the secondary market or for research purposes.

Section 2 presents a basic optimization model as an acceptable method for modeling health-related economic issues (e.g., Brekke et al., 2010; Grassi & Albert Ma., 2011; McKenna et al., 2010; Yaesoubi and Roberts, 2011). Section 3 provides a detailed mathematical analysis of the model, and Section 4 introduces a numerical example with sensitivity analysis. Section 5 concludes the paper and proposes several extensions of the model as directions for further research.

2. The Model

The model considers a service provider such as a hospital that offers cryostorage services to individuals. They sign contracts with the service provider enabling them to store one or more embryos in the provider's facility during a predetermined, finite period of time, *T* (hereinafter also referred to as "the contract period"). We simplify the model by assuming only one unique contract (the one of period *T*) for each consumer. The length of the contract is determined exogenous (e.g., by regulator or social planner). Accordingly, its period can be considered sufficiently long for most couples, however not too long in order not to charge too much. Such a finite contract ensures the mechanism of diminishing unused embryos. Individuals pay a one-time lump sum, K, for each embryo they seek to store, due to fixed costs that are related to initiating the embryo storage. In addition, they pay an annual fee for each embryo, p_0. These annual fees increase or decrease over time, due to the rising holding and operational costs. Since the contact period is pre-determined before the service begins, the entire amount is charged at the beginning of the contract. The objective of the model is to maximize the profits of the storage service provider during the contract period.

2.1 Assumptions and notations

The decision variables are:

K - A one-time lump sum payment for each embryo for the entire period *T*; $K \geq 0$.

p_0 - The base annual fee for storage of a single embryo; $p_0 \geq 0$.

α - The rate at which the annual fee increases or decreases over time.

Since the hospital has modest incentives in operating the storage service, the objective of the suggested model is to maximize the hospital's profits as the service provider. This objective includes maximizing the difference between the service provider's revenues which are a function of the service price set by the provider, and the holding costs.

The service price per embryo unit at time *t* is given by

$$p(t) = p_0 + \alpha t \tag{1}$$

The holding cost per embryo unit at time *t* is given by

$$h(t) = h_0 + \delta t, \tag{2}$$

where h_0 is the initial holding cost for a unit of time, and δ is the rate at which holding costs increase over time ($\delta \geq 0$) due to increasing expected risks associated with longer storage periods.

Since the financial risk of damaged embryos and the probability of damages incurred with stored embryos are very difficult to estimate, our model uses both factors α and δ, thus take into account costs that rise over time, such as insurance premiums that increase as a result of greater risk of failure (Sloan et al., 1989; Zuckerman et al., 1990). The annual market demand function for embryos storage (in addition to the treatments consumption) during the entire contract period is assumed to be linearly decreasing with the three decision variables described above according to

$$D(K, p_0, \alpha) = n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha \tag{3}$$

where the parameters in (1) are defined as follows:

n_0 is the annual demand for storage services (measured in the number of embryos) when all cost components are 0;

β_1 is the responsiveness of D to changes in the one-time lump sum payment ($\beta_1 > 0$);

β_2 is the responsiveness of D to changes in the initial annual fee ($\beta_2 > 0$).

β_3 is the responsiveness of D to changes in the rate of change in the annual fee over time ($\beta_3 > 0$).

The linear dependency between the demand function (3) and the price simplifies the model and is commonly used in the literature (see Xu et al., 2013b; Xu et al., 2013a). Surely, other factors other than price affect demand of such unusual storage service, yet the suggested model focuses on the effect of price in order to generally obtain a decrease of unused embryos and to increase its presence, as explained earlier.

The assumptions of the suggested model are presented below:

Assumption 1. The contract period, *T*, is constant for all customers, is given, and cannot be extended, however can be renewed.

Assumption 2. The demand is linear and decreases with respect to each of the decision variables described above.

Assumption 3. All unused embryos remaining at the end of the contract period are either discarded at no charge or donated to research or to a secondary market by the service provider.

Assumption 4. The holding cost linearly increases with time.

A priori, fixing the contract period prevents nontransparent arrangements between the parties that would extend it beyond the maximum duration. Usually the contract period is long enough for common use and regulated by the authorities (Mohler-Kuo et al., 2009). A short contract period lessens the probability of damaging the embryos. The third assumption in our model enforces social benefits such as decreasing the quantity of unused embryos, and increasing the potential availability of embryos for the secondary market. Assuming a constant (over time) price-dependent demand rate is commonly assumed in literature. A more complex demand function is not assumed in order to simplify the model analysis.

2.2 Model formulation

The profit gained during the contract period, $\prod = TR - TC$, includes the two components of total revenues, TR; and holding costs, TC. The objective function $\pi(K, p_0, \alpha)$ of the hospital that is the service provider is defined as the profit for a unit of time during the contract period. Thus the problem is formulated as

$$\underset{K,P_0,\alpha}{Max}\pi = \frac{1}{T}\left[D(K, p_0, \alpha)\left(K + \int_0^T \big((p_0 + \alpha t) - (h_0 + \delta t)\big)\, dt \right)\right] \quad (4)$$

In order to prevent individuals from possibly obtaining a better price by suggesting division of the contract period into several smaller time segments, the service provider must determine a sufficiently large initial payment per entry, K. This requirement is represented by

$$K + \int_0^T p(t)dt \le 2K + \int_0^{T_1} p(t)dt + \int_{T_1}^T p(t - T_1)dt, \forall T_1 \in (0, T).$$

By definition of $p(t)$,

$$K + \int_0^T (p_0 + \alpha t)dt \le 2K + \int_0^{T_1} (p_0 + \alpha t)dt + \int_{T_1}^T [p_0 + \alpha(t - T_1)]dt, \forall T_1 \in (0, T)$$

Or,

$$K + \alpha T_1^2 - \alpha T T_1 \ge 0. \quad (5)$$

Inequality (5) depends on parameter T_1, which is not a parameter of the problem but is provided for illustration purposes only. We conclude that when the service provider enables individuals to divide the contract period into shorter time segments, they have the financial incentive to do so. Thus, in order to lessen this motivation that also results in additional bureaucratic procedures and overhead costs, we seek the conditions under which any choice of T_1 is not cost beneficial. Therefore condition (5) is always valid. By taking the first order condition, we find that the value of T_1 that optimizes the left-hand side of the inequality is equal to $T/2$. The following proposition assists the service provider in determining the one-time lump sum payment, considering its unwillingness to divide the contract period.

Proposition 1. If $\alpha \geq 0$, then $\alpha T^2/4$ is a lower bound on the one-time lump sum payment.

Proof.

In case $\alpha > 0$ the second order derivative of the left-hand side of (5) results in 2α. Considering that $\alpha > 0$ we conclude that $T_1 = T/2$ is a global minimum. After substituting this point in the left-hand side of (5), we find that $\alpha T^2/4$ is a lower bound on the one-time lump sum payment. In case, $\alpha = 0$, (5) always holds where the lower bound is 0. Thus, the claim is true. □

In conclusion, in order that the service provider can offer a fixed contract period which cannot be divided, the one-time lump sum payment must comply with the following constraint

$$K \geq \alpha T^2/4 \tag{5a}$$

Thus, the detailed optimization problem is

$$\underset{K,P_0,\alpha}{Max}\,\pi = \frac{1}{T}\left[d(K, p_0, \alpha)\left(K + \int_0^T \big((p_0 + \alpha t) - (h_0 + \delta t)\big)dt\right)\right] \tag{6}$$

s.t

$$n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha \geq 0 \tag{6.1}$$

$$p_0 \geq 0 \tag{6.2}$$

$$K \geq \alpha T^2 / 4 \qquad (6.3)$$

Problem (6) with constraints (6.1)-(6.3) is classified as a nonlinear programming problem. In the following section we mathematically analyze the problem and provide an optimal solution. Moreover, we prove that the optimal solution never allows the possibility that all decision variables are simultaneously positive.

3. Mathematical analysis

We first define a Lagrangian function and then the first order necessary conditions for optimization:

$$L(K, p_0, \alpha, \lambda) = \frac{1}{T}\left[\left(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha\right)\left(K + p_0 T - h_0 T + \frac{1}{2}(\alpha - \delta)T^2\right)\right] - \lambda_1\left(\alpha \frac{T^2}{4} - K\right) + \lambda_2\left(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha\right) + \lambda_3 p_0 \qquad (7)$$

The first order condition (FOC) for local maximization is given by the solution to the following system of equations, if there exist non negative scalars $\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0$ and decisions variables p_0, α, and K which comply with (6.1)-(6.3).

$$\frac{\partial L}{\partial \alpha} = -\frac{\beta_3}{T}(K + p_0 T - h_0 T + \frac{1}{2}(\alpha - \delta)T^2) + \frac{1}{2}T(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) - \frac{\lambda_1 T^2}{4} - \lambda_2 \beta_3 = 0 \qquad (7.1)$$

$$\frac{\partial L}{\partial K} = -\frac{\beta_1}{T}(K + p_0 T - h_0 T + \frac{1}{2}(\alpha - \delta)T^2) + \frac{1}{T}(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) + \lambda_1 - \lambda_2 \beta_1 = 0 \qquad (7.2)$$

$$\frac{\partial L}{\partial p_0} = -\frac{\beta_2}{T}(K + p_0 T - h_0 T + \frac{1}{2}(\alpha - \delta)T^2) + (n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) - \lambda_2 \beta_2 + \lambda_3 = 0 \qquad (7.3)$$

$$\lambda_1(\alpha \frac{T^2}{4} - K) = 0 \qquad (7.4)$$

$$\lambda_2(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) = 0 \qquad (7.5)$$

$$\lambda_3 p_0 = 0 \qquad (7.6)$$

In order to simplify tracking (7.1)-(7.6) we define the following:

$$X \equiv \frac{1}{T}\left(K + p_0 T - h_0 T + \frac{1}{2}(\alpha - \delta)T^2 \right)$$

$$Y \equiv n_0 - \beta_1 k - \beta_2 p_0 - \beta_3 \alpha$$

According to (7.5), in case $\lambda_2 > 0$ the demand for a frozen embryo is zero and the objective is not profitable. To simplify the analysis we are interested only in the case that the provider gains positive profit. Thus, we are left with four cases to be examined (i.e., $\lambda_2 = 0$). The mathematical details of the solution method are presented in appendix A. Based on the above analysis we introduce the following proposition:

Proposition 2. Problem (6) obtains its maximal value in one of the following regimes:

$$\alpha^* = \frac{(T\delta + 2h_0)(\beta_1 T^2 + 4\beta_3) + 6n_0 T}{3T(\beta_1 T^2 + 4\beta_3)}, K^* = \frac{T((T\delta + 2h_0)(\beta_1 T^2 + 4\beta_3) + 6n_0 T)}{12(\beta_1 T^2 + 4\beta_3)}, p_0^* = 0 \quad (8.1)$$

$$\alpha^* = 0, K^* = 0,\ p_0^* = \frac{n_0}{2\beta_2} + \frac{h_0}{2} + \frac{\delta T}{4} \quad (8.2)$$

From Proposition 2 we observe that both the optimal one-time lump sum payment K^* and the rate of change in the annual fee α^* are not negative. We conclude that in order to maximize the profits, the service provider should set a nondecreasing price over time (i.e., $\alpha \geq 0$). An increasing price over time along a finite selling horizon is not commonly presented in economic literature. Increasing the price with time along a finite selling horizon is also suggested by Herbon (2014) for inventoried products and by You (2006) for service products.

By defining $\psi(\beta_1, \beta_3, T, h_0, \delta) \equiv \frac{(T\delta + 2h_0)(\beta_1 T^2 + 4\beta_3) + 6n_0 T}{3T(\beta_1 T^2 + 4\beta_3)}$, solution (8.1) is presented in a shorter form as

$$\alpha^* = \psi(\beta_1, \beta_3, T, h_0, \delta), K^* = \frac{\psi(\beta_1, \beta_3, T, h_0, \delta)T^2}{4},\ p_0^* = 0 \quad (9)$$

By substitution of (8.2) and (9) into objective (6), we accordingly obtain π_1^* (One-Part Tariff scheme) and π_2^* (Two-Part Tariff scheme).

where

$$\pi_1^* = \frac{(-2n_0 + 2h_0\beta_2 + \delta T\beta_2)^2}{16\beta_2} \qquad (10)$$

$$\pi_2^* = \frac{(-4n_0 + \beta_1\psi T^2 + 4\beta_3\psi)(-3\psi T + 2T\delta + 4h_0)}{16} \qquad (11)$$

Theorem 1.

(a) The optimal solution of problem (6) is classified as a One-Part or Two-Part Tariff scheme.

(b) Problem (6) has a unique optimal solution. If $\pi_1^* > \pi_2^*$ then (8.2), i.e., the One-Part Tariff scheme is optimal with objective (10), otherwise (9), i.e., the Two-Part Tariff scheme is optimal with objective (11).

Proof.

(a) Immediate, due to Proposition 2.

(b) Immediate, according to definition of optimality. □

The first claim in Theorem 1 implies that the three decision variables are never all positive simultaneously.

4. Numerical study

In this section we numerically demonstrate the applicability of the suggested model and its implications. Consider a hospital as an embryo cryostorage service provider where the maximal annual market demand for storage services is estimated from historical data by 1500 embryos (see Table 1).

4.1 Optimal pricing policy

In this subsection we exemplify the validity of the two possible optimal pricing schemes as theoretically been proved in proposition 2. Consider the following data presented in Table 1:

Table 1. List of parameters.

Parameter	T	δ	n_0	h_0	β_1	β_2	β_3
	6	0.9	1500	300	0.5	0.5	0.5

The values of the three decision variables and the objective obtained by (9) and (8.2) are:

(a) $\alpha = 183.63, K = 1652.70, p_0 = 0, \pi_2 = 3.05 \cdot 10^5$.

(b) $\alpha = 0, K = 0, p_0 = 1651.35, \pi_1 = 9.09 \cdot 10^5$.

We can see that according to the set of parameters, above, solution (b) is the optimal solution. Consider the following table:

Table 2. List of general parameters.

Parameter	T	δ	n_0	h_0	β_1	β_2	β_3
	5	0.1	1500	500	0.1	0.5	0.5

Decision variables and objective value obtained by (9), (8.2) and (16) are:

(a) $\alpha = 733.36, K = 4583.54, p_0 = 0, \pi = 1.52 \cdot 10^6$

(b) $\alpha = 0, K = 0, p_0 = 1750.12, \pi = 7.81 \cdot 10^5$

According to the set of parameters, above, solution (a) is optimal. The two numerical examples comply with Proposition 2. The above examples also confirm that all three price variables initially assumed have the possibility to realize.

4.2 Sensitivity analysis

In this section we examine the effect of changing some of the key parameters on the optimal solution and objective. Consider the following table:

Table 3. List of general parameters.

Parameter	T	δ	n_0	h_0	β_1	β_2	β_3
	5	0.1	1500	500	0.5	0.5	0.5

The optimal solution under the setting, above, is

$\alpha^* = 0, K^* = 0, p_0^* = 1750.13, \pi^* = 781{,}093$.

Figure 1 presents the effect of changing the responsiveness of demand D to changes in the one-time lump sum payment, β_1. According to Figure 1, the provider profit decreases significantly with the responsiveness of the demand to changes in the one-time lump sum payment , β_1. Note that at approximately at $\beta_1 = 0.25$, the curve shows a switching point where the derivative is not continuous, after which the profit remains fixed. We attribute this fact to the point in which the optimal pricing scheme changes from Two-Part Tariff into One-Part Tariff, whereas the demand no longer depends on the one-time lump sum payment.

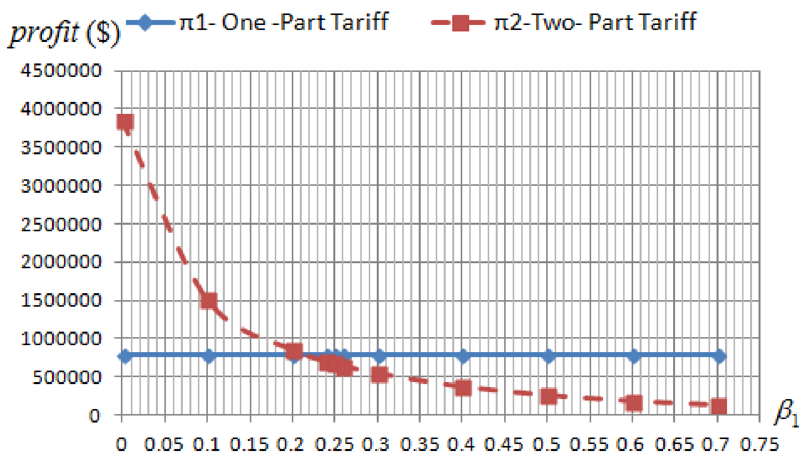

Figure 1. Optimal profit of the service provider for various values of β_1

Figure 2 presents the effect of changing the responsiveness of demand D to changes in the rate of change in the annual fee over time, β_3 . Within the range of the observed values, the optimal pricing scheme is the One-Part Tariff policy, thus the optimal profits are not sensitive to changes in the initial fee β_3 . We attribute this result to the fact that when applying the One-Part Tariff scheme the demand is independent of the rate of change in the annual fee over time.

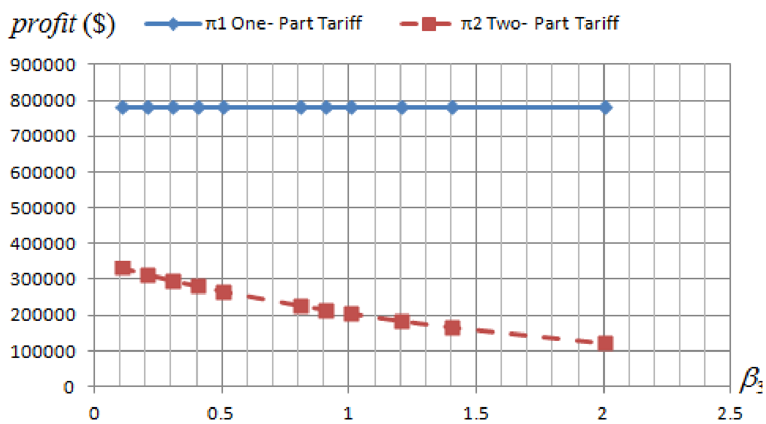

Figure 2. Optimal profit of the service provider for various values of β_3

The observed insensitivity of the demand D to changes in the rate of change in the annual fee over time, β_3 has a practical implication regarding implementation. The service provider does not have to rely on accurate (and costly) estimations of this parameter. Even relatively inaccurate estimation of β_3 will not practically alter expected profits.

The suggested model assumed a pre-determined contract period, T and identicall to all. Figure 3, below, presents the effect of changing the contract period, T. Behavior similar to that in Figure 2 is observed when the contract period, T is changed. Unlike the sensitivity to β_3, the optimal profit in Figure 3 is only approximated by a constant value along the entire searching domain (see (10)) (i.e., the blue curve is nearly horizontal). Results presented in Figure 3 imply that, at least from the service provider perspective, offering a flexible contract (e.g., individuals signing it choose its length) would not alter her optimal profits.

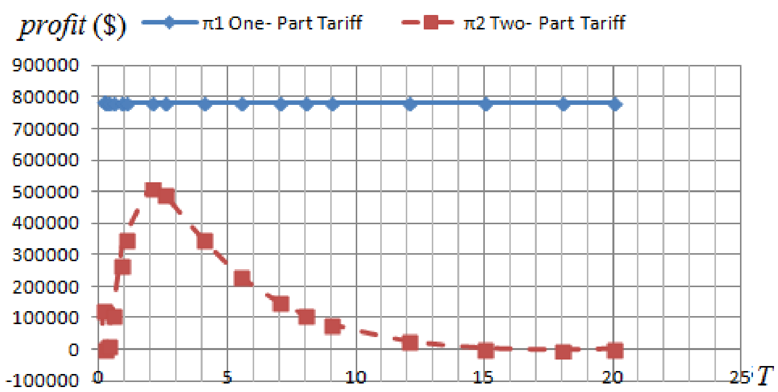

Figure 3. Optimal profit of the service provider for various values of T

Figure 4 presents the effect of changing the initial holding cost for a unit of time, h_0. According to Figure 4, the provider's profit increases with the responsiveness of the demand to changes in the initial holding cost for a unit of time, h_0. Note that at approximately at $h_0 = 1200$, the curve shows a switching point where the derivative is not continuous. We attribute this fact to the point in which the optimal pricing scheme changes from One-Part Tariff into Two-Part Tariff. A possible explanation of the increase in optimal profits is that in both optimal pricing schemes, such as in

(K^*, α^*, p_0^*), the optimal prices increase more than the increase in the initial holding cost for a unit of time, h_0 ,that is the response of the service provider.

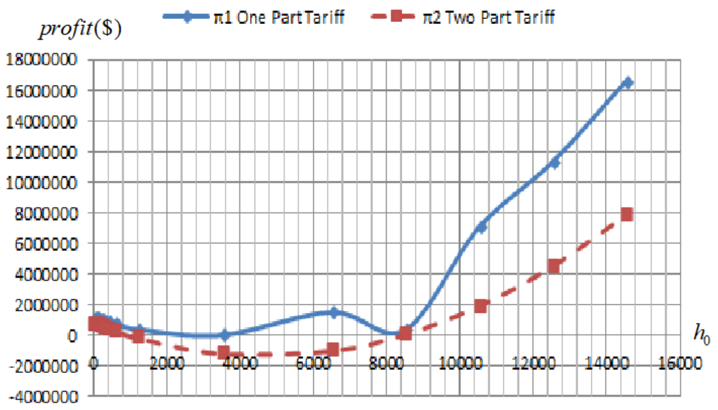

Figure 4. Optimal profit of the service provider for various values of h_0

Figure 5 presents the effect of changing the rate at which holding costs increase over time, δ. Within the range of the observed values, the optimal pricing scheme is the One-Part Tariff policy. However, within the scope of realistic values of parameter δ (e.g., $\delta \in [0,50]$) the optimal profit slightly decreases under changes in δ.

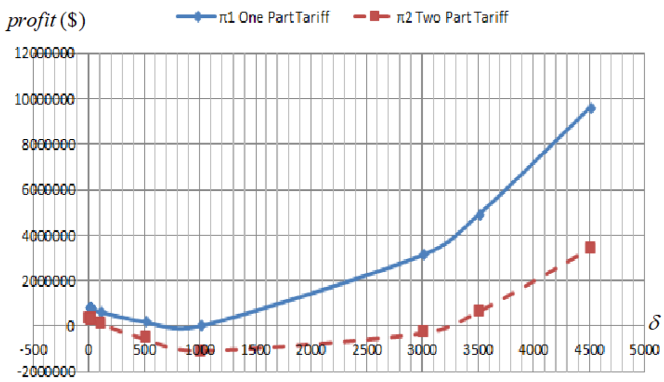

Figure 5. Optimal profit of the service provider for various values of δ

4.3. Real- data comparison

In order to demonstrate the significance of the suggested model, we exemplify the model's optimal solution for several real-data sets. For the case of England, we take the data from Concept Fertility Clinic in London and isolate the embryo freezing fee from all other cryogenic services (e.g., egg freezing, sperm freezing, egg thawing, etc.) According to their price list, they charge an annual frozen embryo fee of $p_0 = \$365$. For the case of Florida we take the data from Fertility Institute and isolate the embryo freezing fee from the IVF egg freezing program. They charge an

annual egg cryopreservation storage fee of $p_0 = \$360$. In the case of California and as suggested by Frozen Egg Bank, the annual charge is $p_0 = \$275$, while in Israel the service for storage of frozen embryos is free of charge. In order to objectively compare the different IVF systems, we use identical parameters setting presented in Table 4.The optimal solution for that setting is
$\alpha^* = 0, K^* = 0, p_0^* = 1750.13, \pi^* = 781{,}093$.

Table 4. List of general parameters.

Parameter	T	δ	n_0	h_0	β_1	β_2	β_3
	5	0.1	1500	500	0.5	0.5	0.5

For the four abovementioned countries and states, Table 5 shows the potential increase in the provider's profit and the potential decrease in unused embryos when the suggested model is utilized.

The pricing scheme practiced by the service provider of country or state *j* is denoted by $(K^j, p_0{}^j, \alpha^j)$, and the optimal one according to the developed model is denoted by $(K^*, p_0{}^*, \alpha^*)$. Accordingly, we denote the potential increase in the provider's profit $\Delta\pi_j$ and the potential decrease in annual unused embryos, ΔI_j as presented below.

$$\Delta\pi_j = \pi(K^*, p_0{}^*, \alpha^*) - \pi(K^j, p_0{}^j, \alpha^j) \qquad (12)$$

$$\Delta I_j = d(K^j, p_0{}^j, \alpha^j) - d(K^*, p_0{}^*, \alpha^*) \qquad (13)$$

Table 5. Potential benefit for several service providers in using the suggested model

Country or state (*j*)	$\pi(K^j, p_0{}^j, \alpha^j)$	$\Delta\pi_j(\$)$	$d(K^j, p_0{}^j, \alpha^j)$	ΔI_j
England	-178,191	959,284	1317.5	692.57
Florida	-185,130	966,223	1320	695.07
California	-306903	1,087,996	1362.5	737.57
Israel	-750,375	1,531,468	1500	875.07

While the growing scale of embryo storage causes an increasing quantity of unused embryos, the suggested model reduces the quantity of unused embryos in storage by about 50% and significantly increases the profits of the service provider. These results present greater incentive for the service provider to continue offering this important service, while significantly diminishing the difficulties posed by storage of unused embryos.

5. Conclusions

5.1 Summary and conclusions

We addressed the problem of maximizing the profit of an embryo cryostorage service provider, for a case in which there is a time-limited pricing contract between the service provider and the individuals seeking cryostorage services. The problem is formulated as a nonlinear programming problem, and the optimal policy is derived through mathematical analysis. It is shown that for any given set of parameters there is a unique global solution. The pricing policy excludes the possibility of dividing the contract period into segments. Thus we computed the lower bound on the one-time lump sum payment. It is shown that there are no instances including all decision variables of the optimal pricing policy. Thus there is no Three-Part Tariff, while the optimal annual base fee for storage of a single embryo never decreases over time. Such a pricing policy motivates individuals to decrease demand and eventually to decrease unused embryos.

Through a numerical example it is shown that there is a significant potential for increasing the provider's profits and at the same time for reducing the quantity of unused embryos in storage by approximately 50%. Interestingly, according to the numerical example, among the three pricing decision variables only the responsiveness of D to changes in the one-time lump sum payment (i.e., β_1) has an effect on the optimal pricing regime.

The suggested model has several managerial implications. The provider now has a greater incentive to continue offering this important service, while significantly diminishing the difficulties posed by unused embryos such as possible legal proceedings and financial penalties. The suggested model assists the service provider in determining the optimal fees charged to individuals, while it decreases the incentive for individuals to keep unused embryos. In making this storage service available, the social planner decreases the quantity of unused embryos while maintaining the option of embryo donation either for the secondary market or for research purposes. The suggested model assumes a fixed contract length, however at least from the service provider perspective, the numerical example shows that a flexible contract (e.g., individuals signing it choose its length) would not alter her optimal profits. In order to maintain the advantages of limited storage periods discussed earlier she can offer couple flexible contracts, yet not exceeding *T*.

The obtained result, under which the optimal profit increases with the initial holding cost for a unit of time, is probably surprising to some. An interesting application of this result may carry in the case that the service provider is aware of this outcome. In particular, she can deliberately increase the value of this parameter beyond its real value, prior to inserting it to the optimization model. Such strategy is expected to increase her profit, even more than is obtained in the numerical example, without any additional investment. This point indicates the need to model the problem discussed in this paper, also with an addition player that is an active participation of the regulator to restraining the prices the service provider can determine.

5.2 Discussion and extensions of the model

Our model can be extended in several research directions:

(a) Given the important objective of minimizing the quantity of unused embryos remaining at the end of the contract period, we suggest incorporating a disposal charge in the model. Individuals who do not use all their stored embryos will have to pay the provider to discard such embryos at the end of the contract period.

(b) Instead of having to discard unused embryos, individuals might be given the option to participate in a secondary market for embryos. This scenario would create two interdependent markets. When the contract period ends, individuals in the primary market will sell the unused embryos to the service provider at a predetermined price. The service provider will then sell some or all of the embryos to individuals in the secondary market, in which demand is external and price-dependent. This process can take place directly or indirectly through a mediator. According to the model, the proportion of unused embryos that are designated for disposal and the complementary maximal proportion of unused embryos designated for sale are given. This mechanism has the potential to increase donations to second market.

(c) A generalization of extension (b) is to consider the economic possibility that pricing decisions made by the service provider will impact the number of individuals that choose to discard their unused embryos and the number that agree to either donate them to the secondary market or sell them to the service provider.

(d) A major postulate of the suggested model is a decision making that pursues profit maximization. Since the regulator has also the incentive to solve the issues discussed in this paper (i.e., increase social welfare), including the guarantee of embryo storage service by subsidizing low-income couples is expected. Developing a model in which the regulator is another active player can be considered for future possible research.

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Appendix A

(a) $\lambda_1 = \lambda_3 = 0$

The FOC reduces to:

$$-\beta_3 X + \frac{TY}{2} = 0 \tag{A.1}$$

$$-\beta_1 X + \frac{Y}{T} = 0 \tag{A.2}$$

$$Y = \beta_2 X \tag{A.3}$$

In the case that $Y = 0$, the demand as well as the profit is zero. We analyze the case where $Y > 0$. Since $\beta_1 > 0, \beta_2 > 0, \beta_3 > 0$, according to assumption 2 we conclude that $X > 0$. In order to comply with (A.1-A.3) the condition $\beta_2 = \beta_1 T = \frac{2\beta_3}{T}$ must hold. Since the demand pattern is exogenous, such a relation between the parameters is not guaranteed. For this particular condition the demand function is $d(K, p_0, \alpha) = n_0 - \beta_1 (K + p_0 T + \frac{\alpha T^2}{2})$. This demand function is inconsistent with the definition of the problem as it obtains positive values when the planning time is zero. We conclude that even this particular condition is excluded, and thus we exclude this case from obtaining an optimal solution.

(b) $\lambda_1 > 0, \lambda_3 = 0$

The FOC reduces to:

$$-\beta_3 X + \frac{TY}{2} - \frac{\lambda_1 T^2}{4} = 0 \tag{A.4}$$

$$-\beta_1 X + \frac{Y}{T} + \lambda_1 = 0 \tag{A.5}$$

$$Y = \beta_2 X \tag{A.6}$$

$$\frac{\alpha T^2}{4} - K = 0 \tag{A.7}$$

Omitting the possibility that $Y = 0$ which is a nonprofitable scenario, and since β_2 cannot be 0 due to Assumption 2, with (A.6) we are left with the only option of $Y > 0$ and $X > 0$. We will refer to two cases according to (A.7):

(b.1) $\alpha = 0, K = 0$

According to (A.6)

$$p_0 = \frac{n_0}{2\beta_2} + \frac{h_0}{2} + \frac{\delta T}{4} \tag{A.8}$$

(b.2) $\alpha \neq 0, K \neq 0$

Multiplying (A.4) by $\frac{4}{T^2}$ and summing it with (A.5) results in

$$Y = \left[\frac{4\beta_3}{3T} + \frac{\beta_1 T}{3} \right] X$$

.

From (A.4) we obtain

$$\beta_2 = \frac{4\beta_3}{3T} + \frac{\beta_1 T}{3}$$

Substituting $K = \frac{\alpha T^2}{4}$ in demand (3) obtains

$$d(K, p_0, \alpha) = n_0 - \alpha \left(\frac{\beta_1 T^2}{4} + \beta_3 \right) - p_0 \left(\frac{4\beta_3}{3T} + \frac{\beta_1 T}{3} \right)$$

From the above expression, $\beta_1 \frac{T^2}{4} + \beta_3 = \beta_3$ i.e., $\beta_1 = 0$. Such a conclusion contradicts the dependency of the demand function in K when $K > 0$, for example in Assumption 2. Thus we exclude this case from obtaining an optimal solution.

(c) $\lambda_1 > 0, \lambda_3 > 0$

The FOC reduces to:

$$-\beta_3 X + \frac{TY}{2} - \frac{\lambda_1 T^2}{4} = 0 \tag{A.9}$$

$$\lambda_1 = \beta_1 X - \frac{Y}{T} \tag{A.10}$$

$$\lambda_3 = \beta_2 X - Y \tag{A.11}$$

$$\frac{\alpha T^2}{4} - K = 0 \tag{A.12}$$

$$p_0 = 0 \tag{A.13}$$

There are two options:

(c.1) $\alpha = 0, K = 0$

Following (A.13), the solution is $p_0 = 0, \alpha = 0, K = 0$.

When $p_0 = 0$, the profit objective $\pi = -\frac{n_0}{T}(h_0 T + \frac{1}{2}\delta T^2)$ is negative. According to Assumption 4 the service provider should shut down the storage service. However, this solution contradicts (A.10), and therefore it is not an optimal solution.

(c.2) $\alpha \neq 0, K \neq 0$

Following (A.12) the solution obtained is $K = \frac{\alpha T^2}{4}, p_0 = 0$ and by substitution of X, Y and (A.10), (A.12), (A.13) into (A.9) we obtain

$$\alpha = \frac{(T\delta + 2h_0)(\beta_1 T^2 + 4\beta_3) + 6n_0 T}{3T(\beta_1 T^2 + 4\beta_3)}, \quad K = \frac{T((T\delta + 2h_0)(\beta_1 T^2 + 4\beta_3) + 6n_0 T)}{12(\beta_1 T^2 + 4\beta_3)}, \quad p_0 = 0 \qquad \text{(A.14)}$$

(d) $\lambda_1 = 0, \lambda_3 > 0$

The FOC reduces to:

$$-\beta_3 X + \frac{TY}{2} = 0 \qquad \text{(A.15)}$$

$$-\beta_1 X + \frac{Y}{T} = 0 \qquad \text{(A.16)}$$

$$\lambda_3 = \beta_2 X - Y \qquad \text{(A.17)}$$

$$p_0 = 0 \qquad \text{(A.18)}$$

Omitting the possibility Y=0, which is a nonprofitable scenario, with (A.17) the only remaining option is $Y > 0$ and accordingly $X > 0$. In order to comply with (A.15)-(A.16) a specific ratio $\beta_3 = \frac{T^2 \beta_1}{2}$ between the parameters β_1 and β_3 should hold. Since the demand is exogenous, this ratio is not guaranteed and thus we exclude this case from obtaining an optimal solution.